On Model Selection Criteria for Climate Change Impact Studies

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Motivation

Setting: For i = 1, 2, ..., n, t = 1, 2, ..., T, we observe a scalar outcome Y_{it} ; in addition, for each i and t, we observe a regressor $W_{it\tau}$ at a higher frequency $\tau = 1, 2, ..., H$.



Large Empirical Literature

- ➢ Weather and Climate Change Impacts (Review: Dell, Jones and Olken, JEL 2014)
 - Agriculture: Deschenes and Greenstone (2007, AER), Schlenker and Roberts (2009, PNAS), Cui (2020, JEEM), Jagnani et al. (forthcoming, EJ), ...
 - Migration: Feng et al. (2010, PNAS), Fan et al. (2018, JAERE), ...
 - Health and Productivity: Deschênes and Greenstone (2011, *AEJ: Applied*), Barreca et al. (2016, *JPE*), Zhang et al. (2018, *JEEM*), Park et al. (2020, *AEJ: Applied*), ...
 - · ...
- ➤ Pollution Impacts

Graff Zivin and Neidell (2012, *AER*), Hanna and Oliva (2015, *JPubE*), Carter et al. (2016, *SciRep*), Metaxoglou and Smith (2020, *AJAE*), He et al. (2020, *JDE*), ...

Motivation

Goals of the Literature

1. Causal Inference

identify the damage/response function that governs the relationship between Y_{it} and the high-frequency regressor $\{W_{it\tau}\}_{\tau=1}^{H}$

2. Prediction of Climate Change Impacts estimate the impact of *future* climate change on outcomes of interest

Empirical Practice

1. construct fixed effects regression model using summary statistics of $T_{it}^k(\{W_{it\tau}\}_{\tau=1}^H)$ as regressors

$$y_{it} = \underbrace{\sum_{k=1}^{K} \beta^{k} T_{it}^{k} (\{W_{it\tau}\}_{\tau=1}^{H})}_{0} + a_{i} + u_{it}$$
(1)

response function

Note: straightforward to accommodate year fixed effects and other control variables

Examples of Summary Statistics

- temperature bins
- various degree day measures
- linear or quadratic function of annual mean

- ...

2. Use the response function to estimate the impact of projected future CC

Motivation

Why Model Selection?

- ➤ While applied researchers typically consider different models and report their results, model selection criteria are still required to choose between the different damage functions to inform policy.
- ≻ Different response functions have different policy implications!
 - different predictions of future climate change impacts
 - different adaptation mechanisms

Example: Predicted Changes in Corn Yields under HadCM3-B1 2015-2050



Which of the above projections should be used to inform policy? To answer this question, some papers use some form of cross-validation.

➤ In this paper, we formally examine the conditions under which Monte Carlo cross-validation and GICs are model selection consistent with the goal of providing formal guidance to practitioners.

This Paper

Roadmap

- \succ Formalize the model selection problem in CC impact studies
- ➤ Provide conditions for model selection consistency of Monte Carlo cross-validation (MCCV) and generalized information criteria (GIC)
- \succ Simulations and two empirical applications illustrate the results

Caveats:

- ➤ model selection consistency as an objective theoretically established trade-off between consistency and risk optimality (Yang 2005)
- ➤ the set of models taken as given: we assume that the researcher's choice of models is informed by scientific literature and/or economic theory (finite-dimensional)
- ➤ fully data-dependent approach to model selection beyond this paper: our results remain relevant since model selection criteria are used to guide tuning parameter choices

Model Selection Problem in Climate Change Impact Studies Setup and Notation

Set of Models Considered: $\{\mathcal{M}_{\alpha}\}_{\alpha=1}^{A}$ where $A < \infty$ For each α , \mathcal{M}_{α}

$$Y_{it} = \underbrace{X_{it,\alpha}'\beta_{\alpha}}_{\text{response function}} + a_{i,\alpha} + u_{it,\alpha}$$

where

-
$$k_{\alpha} \equiv dim(X_{it,\alpha}) = dim(\beta_{\alpha})$$

-
$$\mathcal{W}_{it} \equiv \{W_{it\tau}\}_{\tau=1}^{H}$$

-
$$X_{it,\alpha} = \mu_{\alpha}(\mathcal{W}_{it})$$
, where $X_{it,\alpha}$ is a $k_{\alpha} \times 1$ vector

- $a_{i,\alpha}$ consists of time-invariant unobservables; additional regressors and fixed effects can be readily accommodated

Following the empirical literature, we assume all models considered are linear in the parameters and separable in $a_{i,\alpha}$ and $u_{it,\alpha}$.

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Remarks

While the models under consideration are linear models, this model selection problem is not a simple variable selection problem in linear regression. Two features confirm this: (1) definitions of nested and non-nested models, (2) asymptotic behavior of model selection criteria

Model Selection Problem in Climate Change Impact Studies Definitions

For two models \mathcal{M}_{α} and \mathcal{M}_{γ} , assume wlog $k_{\alpha} < k_{\gamma}$.

- \mathcal{M}_{α} is **nested** in \mathcal{M}_{γ} iff $x_{\omega,\alpha} = R_{\alpha,\gamma}x_{\omega,\gamma}$ for all ω , where $R_{\alpha,\gamma}$ is a $k_{\alpha} \times k_{\gamma}$ non-random matrix,
- \mathcal{M}_{α} and \mathcal{M}_{γ} are **non-nested**, **overlapping** iff \mathcal{M}_{γ} does not nest \mathcal{M}_{α} , but $x'_{\omega,\alpha}\beta_{\alpha} = x'_{\omega,\gamma}\beta_{\gamma}$ for all ω and some $\beta_{\alpha} \in \mathcal{B}_{\alpha}$ and $\beta_{\gamma} \in \mathcal{B}_{\gamma}$,
- \mathcal{M}_{α} and \mathcal{M}_{γ} are strictly non-nested iff they are not nested and $x'_{\omega,\alpha}\beta_{\alpha} \neq x'_{\omega,\gamma}\beta_{\gamma}$ for all $\omega, \beta_{\alpha} \in \mathcal{B}_{\alpha}$ and $\beta_{\gamma} \in \mathcal{B}_{\gamma}$.

Examples: A is nested in QinA and Q, QinA and Q are non-nested overlapping.

- Annual Mean (A): $Y_{it} = \beta_{\alpha} \overline{W}_{it} + a_{i,\alpha} + u_{it,\alpha}$
- Quadratic in Annual Mean (QinA): $Y_{it} = \beta_{\gamma}^1 \bar{W}_{it} + \beta_{\gamma}^2 \bar{W}_{it}^2 + a_{i,\gamma} + u_{it,\gamma}$
- Quarterly Mean (Q): $Y_{it} = \sum_{j=1}^{4} \beta_{\delta}^{j} \bar{W}_{it}^{Q_{j}} + a_{i,\delta} + u_{it,\delta}$

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Comparison with Variable Selection in Linear Models

- \succ Nested: The regressors in \mathcal{M}_{α} is a subset of regressors in \mathcal{M}_{γ} , i.e. elements in $\mathcal{R}_{\alpha,\gamma}$ are either zero or one.
- ≻ Non-nested, overlapping: $x_{\omega,\alpha}$ and $x_{\omega,\gamma}$ include a common subset of regressors.

Consistency of Model Selection Criteria Model Selection Criteria

 \succ MCCV

- MCCV-p: fixed training-to-full sample ratio
- MCCV-Shao: vanishing training-to-full sample ratio (Shao 1993)

$$\succ \text{ GICs: } GIC_{\alpha,\lambda_{nT}} = -n(T-1)\log(MSE) - \lambda_{nT}k_{\alpha}$$

$$- MSE = \sum_{i=1}^{n} \sum_{t=1}^{T} (\tilde{y}_{it} - \tilde{x}'_{it,\alpha}\hat{\beta}_{\alpha})^{2} / (nT), \lambda_{nT} \text{ is the penalty term for the dimension of the model}$$

$$- \text{ Special cases:}$$

$$AIC (\lambda_{nT} = 2)$$

$$BIC (\lambda_{nT} = \log(nT))$$

$$SW_{1} (\lambda_{nT} = \sqrt{nT \log(\log(nT))}) \text{ and } SW_{2} (\lambda_{nT} = \sqrt{nT \log(nT)}), \text{ proposed by Sin and White (1996)}$$

In the following, we examine the model selection consistency of the above criteria.

Consistency of Model Selection Criteria

Summary of Theoretical Results

MCCV (Extending Shao 1993):

Assuming sufficient regularity conditions as well as the following assumptions

- 1. (DGP) For i = 1, 2, ..., n, t = 1, 2, ..., T, $Y_{it} = X_{it,\star}\beta_{\star} + a_{i,\star} + u_{it,\star}$, where $u_{it,\star}|\mathcal{W}_{i1}, \ldots, \mathcal{W}_{iT} \xrightarrow{i.i.d.} (0, \sigma^2)$ across i and t. For some $\alpha = 1, \ldots, A$, $X_{it,\star} = R_{\star,\alpha}X_{it,\alpha}$.
- 2. (Training/Testing Sample Ratios) $n_v/n \to 1$ and $n_c = n n_v \to \infty$, $b^{-1}n_c^{-2}n^2 \to 0$,

$$\Rightarrow P(\hat{\mathcal{M}}_{CV} = \mathcal{M}_*) \to 1 \text{ as } n \to \infty.$$

Key Takeaways

- Traditional implementation of MCCV using large training to full sample ratios are likely to overfit.
- Formal treatment of MCCV requires:
 - (i) homoskedasticity and serial uncorrelatedness in the error term,
 - (ii) the true model is under consideration.

Since both are restrictive, we next examine the conditions under which GICs are model selection consistent.

Model Selection Problem in Climate Change Impact Studies

Summary of Theoretical Results

GICs (Vuong 1989, Sin and White 1996): Assuming Condition 2 in the paper, as $n \to \infty$.

(1) Suppose $E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\alpha};\beta^*_{\alpha}))] = E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\gamma};\beta^*_{\gamma}))]$ and $f(\cdot|\tilde{\mathbf{x}}_{\cdot,\alpha};\beta^*_{\alpha}) = f(\cdot|\tilde{\mathbf{x}}_{\cdot,\gamma};\beta^*_{\gamma})$ hold. Then

$$P(\hat{\mathcal{M}}_{\lambda_{nT}} = \mathcal{M}_{\alpha}) = P\left(GlC_{\alpha,\lambda_{nT}} > GlC_{\gamma,\lambda_{nT}}\right) = P\left(LR_{n}^{\alpha,\gamma} > \lambda_{nT}(k_{\alpha} - k_{\gamma})\right) \to 1,$$

if $\lambda_{nT} \to \infty$.

(2) Suppose $E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\alpha};\beta_{\alpha}^*))] = E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\gamma};\beta_{\gamma}^*))]$ and $f(.|\tilde{\mathbf{x}}_{.,\alpha};\beta_{\alpha}^*) \neq f(.|\tilde{\mathbf{x}}_{.,\gamma};\beta_{\gamma}^*)$ hold, Then

$$P(\hat{\mathcal{M}}_{\lambda_{nT}} = \mathcal{M}_{\alpha}) = (GlC_{\alpha,\lambda_{nT}} > GlC_{\gamma,\lambda_{nT}}) = P\left(\frac{1}{\sqrt{nT}}LR_{nT}^{\alpha,\gamma} > \frac{\lambda_{nT}}{\sqrt{nT}}(k_{\alpha} - k_{\gamma})\right) \to 1,$$

$$\lambda_{nT}/\sqrt{nT} \to \infty.$$

(3) Suppose, without loss of generality, that $E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\alpha};\beta^*_{\alpha}))] > E^0[\log(f(\tilde{Y}_i|\tilde{X}_{i,\gamma};\beta^*_{\gamma}))]$ holds. Then

$$P(\hat{\mathcal{M}}_{\lambda_{nT}} = \mathcal{M}_{\alpha}) = P(G|C_{\alpha,\lambda_{nT}} > G|C_{\gamma,\lambda_{nT}}) = P\left(\frac{1}{nT}LR_{nT}^{\alpha,\gamma} > \frac{\lambda_{nT}}{nT}(k_{\alpha} - k_{\gamma})\right) \to 1,$$

if $\lambda_{nT}/(nT) \to 0.$

Implications

if

- \succ AIC is not model selection consistent.
- \succ BIC is model selection consistent in cases (1) and (3), but not (2), which occurs if none of the models under consideration nest the true model (see Section 3.2.1).
- \succ SW₁ and SW₂ are consistent under all three cases.

Notation: f(.) is the conditional density, β^*_{α} is the probability limit of the fixed effects estimator of the slope coefficient of \mathcal{M}_{α} .

Consistency of Model Selection Criteria

Summary of Baseline Simulation Analysis

Design

DGPs: Annual Mean (A), Quadratic in Annual Mean (QinA), Quarterly Mean (Q)

Models: (1) a set of nested models, (2) a set of nested and non-nested models

Results: Consistent with theoretical analysis...

- ➤ When the true model (DGP) is nested in the set of models under consideration, MCCV-Shao, BIC, SW₁ and SW₂ select the most parsimonious model that nests the true model, whereas AIC and MCCV-p either select the true model or larger models that nest it with high probability.
- \succ When the true model (DGP) is not nested in any of the models under consideration, then only SW₁ and SW₂ are pseudo-consistent, whereas the remaining criteria, including BIC, may overfit.

Since the signal-to-noise ratio of the design can impact the finite-sample performance of model selection procedures (c.f. Hastie et al 2020), we vary the signal-to-noise ratio.

Consistency of Model Selection Criteria Simulation Analysis with Varying Signal-to-Noise Ratio



▶ Details of Sim Design

Note: N denotes the null model with fixed effects only.

Consistency of Model Selection Criteria Simulation Analysis with Varying Signal-to-Noise Ratio



Note: N denotes the null model with fixed effects only.

Consistency of Model Selection Criteria Summary of Simulation Analysis

Key Takeaway for Empirical Practice

When the true model is nested in at least one of the models under consideration:

- \succ For higher SNR levels: BIC, MCCV-Shao and the SW criteria choose it with high probability, whereas AIC and MCCV-p may choose larger models that nest it.
- ≻ For low SNR levels: The SW criteria may "underfit."

Empirical Illustration

Empirical Application I: Temperature and Corn Yields Data: corn sample based on Schlenker and Roberts (2009, *PNAS*), extended to 2015

- This example exhibits a relatively high SNR ($\simeq 60\%$).

Empirical Application II: Temperature and GDP (low SNR) Data: GDP and temperature data from Burke, Hsiang and Miguel (2015, *Nature*)

- This example exhibits a very low SNR (< 1%).

Empirical Application I: Temperature and Crop Yields

Data: corn yields covering 1950-2015 from USDA Quick Stats, weather data from PRISM dataset (Schlenker and Roberts 2009, extended to 2015) **Regression**:

$$\log(Y_{it}) = X'_{it,\alpha}\beta_{\alpha} + \theta_1 P_{it} + \theta_2 P_{it}^2 + \delta_{1,s}t + \delta_{2,s}t^2 + a_i + \epsilon_{it},$$

$$\begin{split} Y_{it}: & \text{ corn yields (bushels/acre) in county } i \text{ in crop year } t \\ X_{it,\alpha} &= \mu_{\alpha}(\mathcal{T}_{it}): \text{ temperature variables constructed based on daily average temperature of the growing season } \mathcal{T}_{it} &\equiv \{T_{it\tau}\}_{\tau=1}^{H} \\ P_{it}: \text{ growing season total precipitation} \end{split}$$

 $\delta_{1,s}, \, \delta_{2,s}$: state-level quadratic trends

Temperature specifications:

- a. No temperature variables
- b. monthly avg temp
- c. 1°C bin (using average daily temp)
- d. 3° step function (sinosudial interpolation of min-max temp before binning)
- e. SHF degree days
- f. one-knot spline
- g. two-knot spline

Empirical Application I: Estimation Results



- ≻ The figure presents the estimation results with pointwise 95% confidence intervals for the one-knot spline, two-knot spline, and 3°C step function.
- ≻ The results for each model is quite similar whether we use the unbalanced or balanced panel.
- ➤ When we compare the different models, they provide very similar response/damage functions despite having a different number of parameters.

Empirical	Application	I:	Model	Selection	Results
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Model	SNR	AIC	BIC	MCCV-p	MCCV-S	SW_1	SW2
Unbalanced panel							
a. no temperature var	10.11%	-333091	-332490	0.07126	0.07369	-299391	-259436
b. monthly avg temp	25.29%	-348417	-347757	0.06443	0.06752	-311455	-267634
c. 1 [°] C daily temp bin	36.76%	-358753	-357792	0.05979	0.06218	-304941	-241143
d. 3 ^o C step function	41.52%	-362864	-362127	0.05768	0.05984	-321554	-272577
e. SHF degree days	38.13%	-361052	-360421	0.05795	0.05990	-325721	-283833
f. one-knot spline	40.82%	-361251	-360630	0.05778	0.05954	-326463	-285220
g. two-knot spline	39.35%	-362102	-361471	0.05744	0.05923	-326771	-284883
Balanced panel							
a. no temperature var	11.42%	-138208	-137895	0.04784	0.04998	-126545	-113340
b. monthly avg temp	30.19%	-145067	-144702	0.04245	0.04585	-131460	-116054
c. 1°C daily temp bin	56.69%	-153185	-152558	0.03530	0.03797	-129858	-103447
d. 3 ^o C step function	64.66%	-155416	-154980	0.03322	0.03549	-139217	-120876
e. SHF degree days	61.94%	-153302	-152962	0.03583	0.03787	-140667	-126361
f. one-knot spline	62.25%	-154706	-154375	0.03310	0.03486	-142395	-128455
g. two-knot spline	56.88%	-154789	-154449	0.03315	0.03500	-142153	-127848

 \succ AIC and BIC choose the 3°C step model, whereas SW₁ and SW₂ select the two- and one-knot spline, respectively. (Cross-validation exercise in SR 2009 selects the 3C step function.)

- ➤ The results are similar whether we consider the balanced or unbalanced sample.
- ➤ The selected models yield similar response functions, though they differ in the number of parameters required to estimate them.
- ➤ These results are consistent with the simulation designs where the true model is contained in the set of models under consideration.
- ➤ Since the selected models yield similar response functions, they also provide very similar climate change projections.

Empirical Application I: Climate Change Projections

Predicted changes in corn yields under HadCM3-B1: 2015-2050



➤ In terms of projections, the 3°C bins and the one-knot and two-knot splines, selected by the different criteria, deliver very similar results.

Details on HadCM3-B1

Empirical Application II: GDP and Temperature

Data: country-level per capita GDP over 1960-2012 for 160 countries (World Bank's World Development Indicators), aggregated country-level annual temperature and precipitation over 1900-2010 (University of Delaware reconstruction)

Regression:

$$\log(Y_{it}) = X'_{it,\alpha}\beta_{\alpha} + \theta_1 P_{it} + \theta_2 P_{it}^2 + \delta_{1,i}t + \delta_{2,i}t^2 + a_i + \mu_t + \epsilon_{it}.$$
 (2)

 Y_{it} represents per capita GDP in country *i* in year *t* $X_{it,\alpha} = \mu_{\alpha}(T_{it})$, where T_{it} is annual temperature P_{it} is annual precipitation

Temperature Specifications:

- a. no temperature variable at all,
- b. simple average,
- c. quadratic,
- d. cubic,
- e. fourth order,
- f. linear spline with knots at every $5^{\circ}\mathrm{C}$ from $0^{\circ}\mathrm{C}$ to $25^{\circ}\mathrm{C},$
- g. linear spline with knots at every 3°C from 0°C to 27°C.
- h. linear spline with one knot determined by minimizing MSE.

Empirical Application II: Estimation Results



- ≻ While the results for most models are consistent across unbalanced and balanced samples, that is not the case for all of them.
- ➤ For the unbalanced panel, the one-knot spline provides a response function that is similar to the quadratic function, whereas for the balanced panel it provides very different results.

Model	$\widehat{R^2}$	SNR	AIC	BIC	MCCV-p	MCCV-S	SW_1	SW_2
Unbalanced panel:								
a. none	0.09%	0.09%	-37166	-34558	0.00542	0.00690	8007	54457
b. simple avg	0.09%	0.09%	-37164	-34549	0.00543	0.00690	8126	54698
c. quadratic	0.46%	0.46%	-37186	-34564	0.00526	0.00674	8222	54915
d. cubic	0.47%	0.47%	-37184	-34556	0.00527	0.00676	8341	55155
e. 4th order	0.47%	0.48%	-37183	-34547	0.00529	0.00679	8460	55395
f. 5°C spline	0.58%	0.58%	-37183	-34527	0.00519	0.00668	8813	56110
g. 3°C spline	0.84%	0.84%	-37192	-34509	0.00515	0.00668	9275	57056
h. one-knot spline	0.52%	0.52%	-37189	-34568	0.00511	0.00659	8218	54911
Balanced panel:								
a. none	0.07%	0.07%	-25160	-23747	0.00424	0.00516	-4387	16503
b. simple avg	0.11%	0.11%	-25160	-23740	0.00423	0.00513	-4293	16691
c. quadratic	0.41%	0.41%	-25172	-23753	0.00416	0.00507	-4306	16678
d. cubic	0.42%	0.43%	-25169	-23736	0.00416	0.00508	-4115	17057
e. 4th order	0.44%	0.44%	-25167	-23729	0.00417	0.00510	-4020	17246
f. 5°C spline	0.43%	0.43%	-25163	-23711	0.00408	0.00501	-3828	17626
g. 3°C spline	0.77%	0.77%	-25169	-23692	0.00402	0.00499	-3461	18370
h. one-knot spline	0.54%	0.54%	-25176	-23750	0.00424	0.00516	-4215	16863

Empirical Application II: Model Selection Results

- ➤ model without any temperature variables chosen by SW criteria for both the unbalanced and balanced panel (potential for "underfitting")
- \succ inconsistent results across samples:
 - unbalanced panel: the one-knot spline is chosen by BIC and both MCCVs, whereas AIC selects the 3° C spline, delivering similar response functions.
 - balanced panel: AIC chooses the one-knot spline, BIC chooses the quadratic model, the MCCV criteria choose the 3° C spline. These models deliver different response functionts, especially the one-knot spline.
- ➤ smoothing splines with a cubic spline basis deliver more consistent results across the two samples in line with the SW criteria ◆ SS Results

Synthesizing Theoretical, Simulation and Empirical Results

- ➤ Rather than reporting the results for a single model selection criterion, the results here suggest that reporting a range of model selection criteria can be informative.
 - If the true model is nested in one of the models under consideration, all criteria should deliver similar response functions, despite their varying number of parameters (e.g. yield-temperature relationship).

≻ Given its relevance for the finite-sample behavior of the model selection criteria, the signal-to-noise ratio should always be reported.

➤ For settings where the models are not supported by the scientific or economic literature, it is important to complement analysis with a fully data-dependent procedure to estimate the response function.

Concluding Remarks

- ➤ This paper formalizes the model selection problem in CC impact studies emphasizing its "nonlinear nature".
- ➤ We provide conditions for model selection consistency via MCCV and GICs in the context of CC impact studies illustrated via simulations and applications.
- \succ The results have several implications for empirical practice:
 - The practice of using MCCV-p with fixed training-to-full sample proportions has a tendency to overfit, which is especially problematic in settings where the models are not grounded in science or economics.
 - While SW_1 and SW_2 are model selection consistent in general, they may underfit in finite samples when the signal-to-noise ratio is low.

Recommendation: report several model selection criteria as well as SNR

- ➤ This paper assumes that the set of models considered is informed by the scientific or economic literature. For settings where such information is not available, fully data-dependent procedures should be employed.
- ➤ Important direction for future work: fully data-dependent procedure with valid post-selection inference

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- \succ Important direction for future work: fully data-dependent procedure with valid post-selection inference

Comments are welcome @ dghanem@ucdavis.edu ! Thank you!

Details of Simulation Design

The following response functions generate Y_{it} for the three DGPs we consider:

- ► Annual Mean (A): $Y_{it} = \overline{W}_{it} + a_{i,\alpha} + u_{it,\alpha}$,
- ▶ Quadratic in Annual Mean (*QinA*): $Y_{it} = 0.2\bar{W}_{it} 0.05\bar{W}_{it}^2 + a_{i,\delta} + u_{it,\delta}$,
- ▶ Quarterly Mean (Q): $Y_{it} = -0.25 \overline{W}_{it}^{Q_1} + 0.75 \overline{W}_{it}^{Q_3} + a_{i,\gamma} + u_{it,\gamma}$.

Temperature Data: We use a random sample of counties from the NCDC temperature dataset for the years 1968-1972 as W_{it} for i = 1, ..., n and t = 1, ..., T, where T = 5.

Unobservables:

-
$$a_i | \mathcal{W}_{i1}, \mathcal{W}_{i2}, \ldots, \mathcal{W}_{i5} \stackrel{i.i.d.}{\simeq} N(0.5 \overline{W}_i, 1)$$
, where $\overline{W}_i = \sum_{t=1}^{T} \sum_{\tau=1}^{H} W_{it\tau} / (TH)$.
- $u_i = (u_{i1}, \ldots, u_{iT}) = \epsilon_i^1 + \epsilon_i^2$, where $\epsilon_i^1 | \mathcal{W}_{i1}, \ldots, \mathcal{W}_{i5}, a_i \stackrel{i.i.d.}{\sim} N(-0.5, \Sigma_1)$ and $\epsilon_i^2 | \mathcal{W}_{i1}, \ldots, \mathcal{W}_{i5}, a_i \stackrel{i.i.d.}{\sim} N(0.5, \Sigma_2)$, with

$$\Sigma_1 = \left(\begin{array}{cccccc} 1 & 0.5 & 0.1 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0.1 & 0 \\ 0.1 & 0.5 & 1 & 0.5 & 0.1 \\ 0 & 0.1 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0.1 & 0.5 & 1 \end{array} \right), \ \Sigma_2 = \left(\begin{array}{cccccccccc} 1 & 0.5 & 0.1 & 0 & 0 \\ 0.5 & 0.75 & 0.5 & 0.1 & 0 \\ 0.1 & 0.5 & 1 & 0.5 & 0.1 \\ 0 & 0.1 & 0.5 & 0.75 & 0.5 \\ 0 & 0 & 0.1 & 0.5 & 1 \end{array} \right)$$

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HadCM3-B1 Scenario

Developed by UK Meteorological Office

Assumptions

- Rapid economic growth as in A1, but with rapid changes towards a service and information economy.
- Population rising to 9 billion in 2050 and then declining as in A1.
- Reductions in material intensity and the introduction of clean and resource efficient technologies.
- An emphasis on global solutions to economic, social and environmental stability.

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Warming: 1.9°C globally, 3°C in North America
Source: https://sos.noaa.gov/datasets/
climate-model-temperature-change-hadley-b1-1870-2100/
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GDP-temperature: Smoothing Spline Results

A. Unbalanced

B. Balanced



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