What Drives Temperature Anomalies?
A Functional SVAR Approach

Yoosoon Chang,[a] J. Isaac Miller,[b] and Joon Y. Park[a,c]

[a]Department of Economics, Indiana University
[b]Department of Economics, University of Missouri
[c]Department of Economics, Sungkyunkwan University

Climate Econometrics Seminar

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Climate Econometrics

"Global warming begins and ends with human activities."

– William Nordhaus (2013), 2018 Nobel Laureate

- Economic activity drives emissions
  (Nordhaus, 1991; *inter alia*)
- Emissions and accumulations influence the climate system
  "A number of studies have applied [econometric] methods... to assess the evidence for a causal link between external drivers of climate and observed climate change.... The advantage... is that they do not depend on the accuracy of any complex global climate model...." (Intergovernmental Panel in Climate Change Fifth Assessment Report, 2013).
- Climate change has economic costs
  (Burke *et al.*, 2015; Hsiang *et al.*, 2017; *inter alia*)

Our Base Model: Structural VAR

Structural VAR (SVAR)

- Workhorse of empirical macroeconomics for decades
- Requires only ...
  - ... simple statistical estimation of a system of equations
  - ... identification based on plausible theory
- Allows dynamic feedbacks in economic systems
- Isolates effects of specific innovations on specific series

Functional SVAR

- Allows distributional dynamic feedbacks
- Isolates temporal effects of structural shocks on spatial distributions

VARs in Climate Science?

Reduced-Form VARs in Climate Science

- IPCC (2013): Statistical time series techniques are useful
- Cointegrated reduced-form VARs have been used extensively: Kaufmann and Stern (2002), Kaufmann *et al.* (2006a,b, 2010), Kaufmann and Juselius (2013, 2016), Pretis (2019)

Structural VARs in Climate Science

- Effects of specific series are of key interest: Especially natural vs. anthropogenic drivers of climate change
- Climate systems have nonlinear dynamic feedbacks
- VARs are useful to identify macroeconomic policy effects, so might they be useful to identify climate policy effects
- SVARs underutilized in climate science

Shocks and Temperature Distribution

We analyze the effects of global economic activity and anthropogenic forcings (greenhouses gases and tropospheric aerosols) net of natural forcings (solar and volcanic activity) on the climate system.

- What are the empirical contributions of shocks in postulated drivers of climate change on global mean temperatures?
- How do shocks in such drivers affect fluctuations in aspects of temperature distribution other than the mean?
- Do shocks to global real economic activity affect the temperature anomaly distribution?
- Which shocks have the largest and/or most permanent effects on variations in temperature distribution?
- Do temperature shocks feedback to economic activity?

Outline of the Rest of the Talk

I. Functional Analysis & Autoregression
- I.A Some Basics of Functional Analysis
- I.B Functional Autoregression (FVAR)

II. Functional SVAR with Temperature Anomalies
- II.A Model & Characteristics of the Data
- II.B Conventional SVAR with Temperature Aggregates
- II.C FSVAR: Impulse Responses
- II.D FSVAR: Forecast Error Variance Decompositions
- II.E FSVAR: Historical Decompositions

III. Some Takeaways

I. Functional Analysis

& Autoregression

Distributional Dynamics

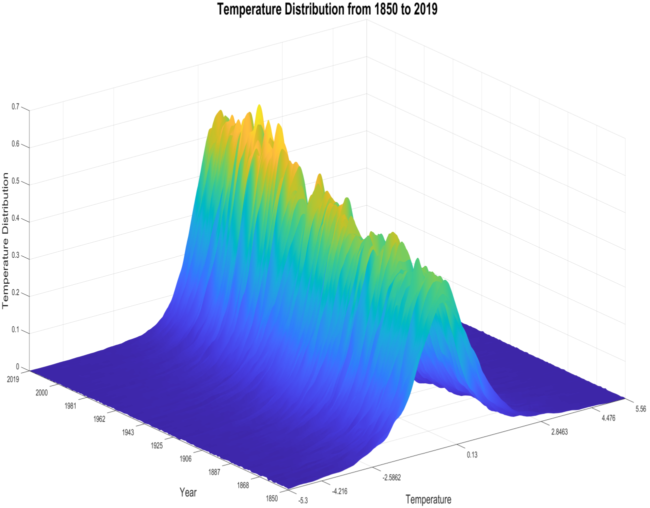
New framework and methodology are introduced to

- analyze distributional dynamics of the time series of global temperature anomaly distributions, and
- study the effects of various natural and anthropogenic shocks to the climate system.

In particular, we aim to disentangle effects on the global distribution of temperature anomalies, net of solar and volcanic activity, of shocks to

- global real economic activity (production),
- accumulation of greenhouse gases, and
- tropospheric aerosols resulting from sulfate emissions.

Densities of Temperature Anomalies



Hilbert-Valued Random Variables

Let

$$w: \ \Omega \to H$$

where H is a Hilbert space. For $w, v \in H$, we denote by $\langle w, v \rangle$ and $\|w\|$ the inner product and norm defined for H.

Hilbert-valued random variables include

- Real random variables: $H = \mathbb{R}$ with
  Euclidean inner product $\langle w, v \rangle = wv$
- Vector-valued random variables:$H = \mathbb{R}^N$ with
  Euclidean inner product $\langle w, v \rangle = w'v = \sum_{i=1}^{N} w_i v_i$
- Function-valued random variables: $H = L^2(\mathbb{R})$ with
  L^2 inner product $\langle w, v \rangle = \int v(s)w(s)ds$

Mean and Variance Operator

The mean $\mathbb{E}w$ of a random variable in H is defined as an element in H satisfying

$$\langle v, \mathbb{E}w \rangle = \mathbb{E}\langle v, w \rangle$$

for all $v \in H$, which exists if $\mathbb{E}\|w\| < \infty$.

For w such that $\mathbb{E}w = 0$, the variance $\mathbb{E}(w \otimes w)$ of w is given by an operator for which

$$\mathbb{E}\langle u, w \rangle \langle w, v \rangle = \langle u, \mathbb{E}(w \otimes w)v \rangle$$

for all $u, v \in H$, which exists if $\mathbb{E}\|w\|^2 < \infty$.

- For a finite dimensional w, $w \otimes w$ reduces to ww', and $\mathbb{E}(w \otimes w)$ reduces to $\mathbb{E}ww'$.

Representation and Implementation

Let $H = L^2(\mathbb{R})$ and (w_t) be a sequence of square integrable random functions. Since $L^2(\mathbb{R})$ is separable, we may write (w_t) as

$$w_t = \sum_{i=1}^{\infty} \langle v_i, w_t \rangle v_i$$

for any orthonormal basis (v_i) of $L^2(\mathbb{R})$.

We use the functional principal component basis to interpret (v_i) as factors and $(\langle v_i, w_t \rangle)$ as factor loadings.

A Mixture of VAR and FVAR

Our subsequent model will consist of

- Two exogenous global variables: natural drivers of climate change
- Three endogenous global variables: a proxy for economic activity, two anthropogenic drivers of climate change
- One endogenous functional variable: density of temperature anomalies

Ignoring the exogenous variables for now, our model includes both scalar variables and functional variables, and we let

- z_t: n-dimensional vector of endogenous global variables
- f_t: an endogenous functional variable

more explicitly.

Product Space

Define

$$\zeta_t = (z_t, f_t)$$

which we regard as a time series of random elements taking values in the product space $\mathcal{H} = \mathbb{R}^n \times H$. We denote by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ the inner product and norm defined for H.

We endow $\mathcal{H} = R^n \times H$ with the usual inner product and norm, $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ and $\| \cdot \|_{\mathcal{H}}$, for the product space, which are given by

$$\langle \zeta, w \rangle_{\mathcal{H}} = z'y + \langle f, g \rangle \quad \text{and} \quad \|\zeta\|_{\mathcal{H}}^2 = z'z + \|f\|^2$$

for $\zeta = (z, f)$ and $w = (y, g)$.

Extended FVAR & Mixture Representation

In the simplest case, let (ζ_t) be generated by a FVAR(1) as

$$\zeta_t = A\zeta_{t-1} + \varepsilon_t,$$

where A is a compact linear operator in $\mathcal{H} = \mathbb{R}^n \times H$, or equivalently by

$$\begin{aligned} z_t &= A_{11} z_{t-1} + A_{12} f_{t-1} + \varepsilon_t^z \\ f_t &= A_{21} z_{t-1} + A_{22} f_{t-1} + \varepsilon_t^f, \end{aligned}$$

where $A_{11} : \mathbb{R}^n \to \mathbb{R}^n$, $A_{12} : H \to \mathbb{R}^n$, $A_{21} : \mathbb{R}^n \to H$ and $A_{22} : H \to H$ are bounded linear operators. Here (ε_t) are random elements in $\mathcal{H} = \mathbb{R}^n \times H$ defined as

$$\varepsilon_t = (\varepsilon_t^z, \varepsilon_t^f)$$

with reduced form errors (ε_t^z) and (ε_t^f) corresponding to (z_t) and (f_t), respectively.

Identification

We define $(n+1)$-dimensional structural shocks (e_t) as $e_t = (e_t^{z\prime}, e_t^f)'$, and let

$$\begin{pmatrix} \varepsilon_t^z \\ \varepsilon_t^f \end{pmatrix} = B \begin{pmatrix} e_t^z \\ e_t^f \end{pmatrix},$$

where

$$B : \mathbb{R}^{n+1} \to \mathcal{H} = \mathbb{R}^n \times H$$

is a bounded linear impact operator. For identification of the structural shocks (e_t), the operator B is specified with restrictions.

Let

$$\Sigma = \mathbb{E}(\varepsilon_t \otimes \varepsilon_t).$$

The operator B is identified if and only if there exists a unique B such that (i) B satisfies the given restrictions, and (ii) $\Sigma = BB^*$ (B^* is the adjoint of B).

Finite-Order Approximation, Part I

We let

$$\text{var}\,(\varepsilon_t^f) = \mathbb{E}\big(\varepsilon_t^f \otimes \varepsilon_t^f\big),$$

and denote by (λ_i, v_i) the pairs of eigenvalues $\lambda_1 > \lambda_2 > \cdots$ and corresponding eigenvectors $v_1, v_2, \ldots$ of var (ε_t^f). Furthermore, we let

$$H_m = \text{span}\,\big\{v_1, \ldots, v_m\big\},$$

and let Π_m be the projection on H_m.

We approximate B by $B_m : \mathbb{R}^{n+1} \to \mathcal{H}_m = \mathbb{R}^n \times H_m$ defined by

$$\begin{pmatrix} \varepsilon_t^z \\ \Pi_m(\varepsilon_t^f) \end{pmatrix} = B_m \begin{pmatrix} e_t^z \\ e_t^f \end{pmatrix}$$

where B_m is an element of the product basis given by the standard basis of R^n and $v_1, \ldots, v_m$ of H.

Finite-Order Approximation, Part II

Define

$$\pi_m : v \mapsto \left(\begin{array}{c} \langle v_1, \Pi_m v \rangle \\ \vdots \\ \langle v_m, \Pi_m v \rangle \end{array} \right)$$

for any $v \in H$, so that π_m is an isometry between H_m and $\mathbb{R}^m$.

Using the isometry, we may write

$$\left(\begin{array}{c} \varepsilon_t^z \\ \pi_m(\varepsilon_t^f) \end{array} \right) = B_m \left(\begin{array}{c} e_t^z \\ e_t^f \end{array} \right)$$

where B_m is redefined to be an $(n+m) \times (n+1)$ matrix.

Note: $(\varepsilon_t^{z\prime}, \pi_m(\varepsilon_t^f)')'$ is an $(n+m)$-vector of fitted residuals from a reduced-form VAR using $\pi_m(f_t)$.

Implementation

We write

$$\text{var}\begin{pmatrix} \varepsilon_t^z \\ \pi_m(\varepsilon_t^f) \end{pmatrix} = \sum_{i=1}^{\infty} \mu_i(w_i w_i'),$$

where (μ_i, w_i) are the pairs of eigenvalues $\mu_1 > \cdots > \mu_{n+m}$ and corresponding eigenvectors $w_1, \ldots, w_{n+m}$ of var $(\varepsilon_t^{z\prime}, \pi_m(\varepsilon_t^f)')'$, and define

$$\Sigma_m = \sum_{i=1}^{n+1} \mu_i(w_i w_i'),$$

which is an $(n+m)$-dimensional square matrix of rank $(n+1)$.

Now we may find B_m such that $\Sigma_m = B_m B_m'$. For B_m to be unique, we need to have $n(n+1)/2$ restrictions – i.e., the number of restrictions required to just identify an SVAR consisting of $(n+1)$ variables. Specifically, we find a matrix B_m that minimizes $||\Sigma_m - B_m B_m'||$.

II. Functional SVAR with

Temperature Anomalies

Time Series of Interest

Consider the vectors

$$x_t = (S_t, V_t)' \text{ (strictly exogenous)}$$

and

$$z_t = (Y_t, G_t, A_t, T_t)', \text{ (endogenous)}$$

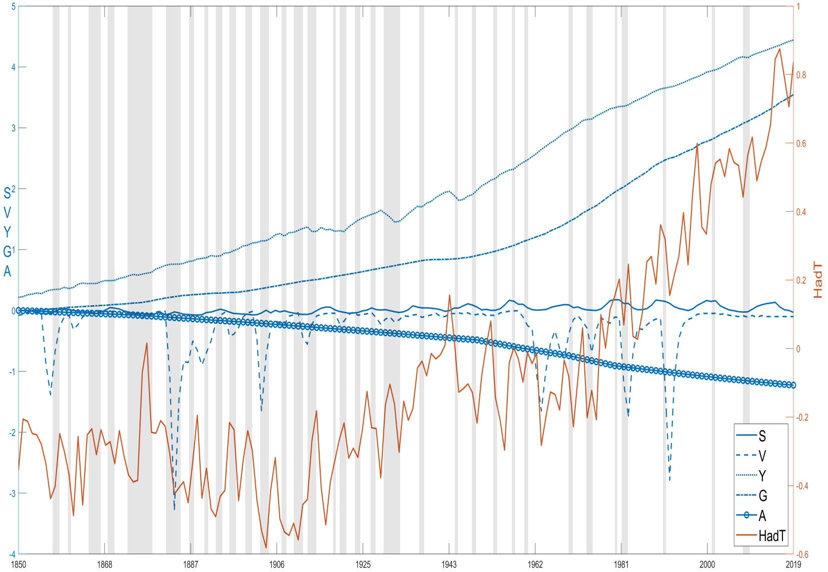
where

- S : Solar activity (W/m^2),
- V : Stratospheric aerosols from volcanic activity (W/m^2),
- Y : Global economic production (log 2010 US$T),
- G : Greenhouse gas concentration (W/m^2),
- A : Tropospheric aerosols and land use (W/m^2), and
- T : Temperature anomaly ($^\circ$C).

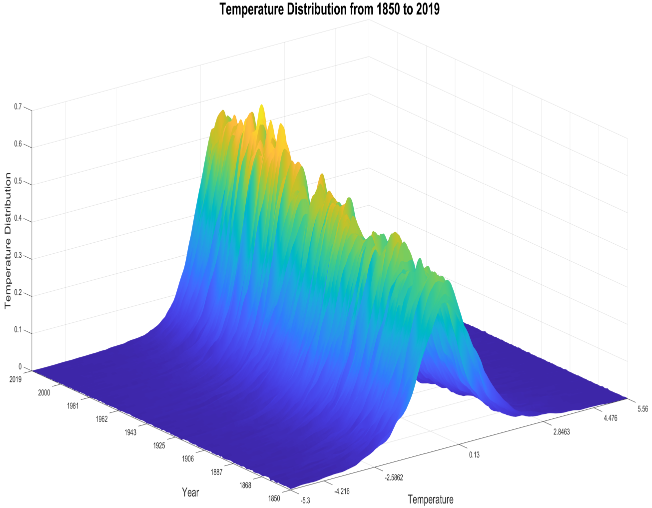
Stuctural VAR Models and Data

- Conventional Aggregate Structural VAR (SVAR)
  - Two exogenous global variables: S, V
  - Four endogenous global variables: $Y, G, A, HadT$
- Functional Structural VAR (FSVAR)
  - Two exogenous global variables: S, V
  - Three endogenous global variables: Y, G, A
  - One endogenous functional variable: $FTemp$, demeaned spatial densities of temperature anomalies
- Data Sources (1850-2019):
  - T : HadCRUT.5.0.1.0 (Morice *et al.*, 2021)
  - S, V, G, A : Hansen *et al.* (2017), updated to 2019 using data from NOAA and NASA
  - Y : World Bank data back to 1960, annualized back to 1850 using Maddison Project Database 2020

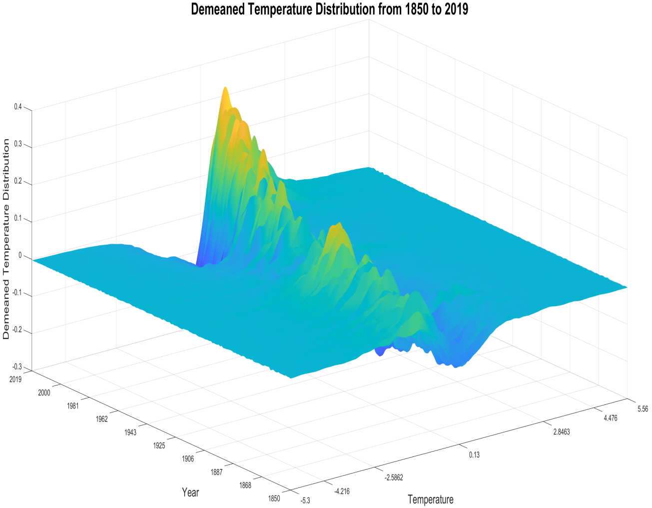
Annual Time Series



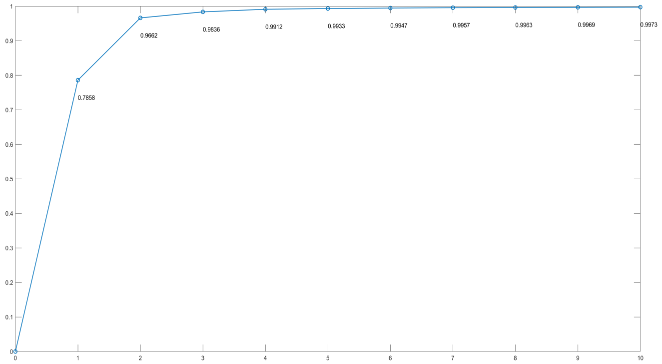
Densities of Temperature Anomalies



Temporally Demeaned Densities of Anomalies

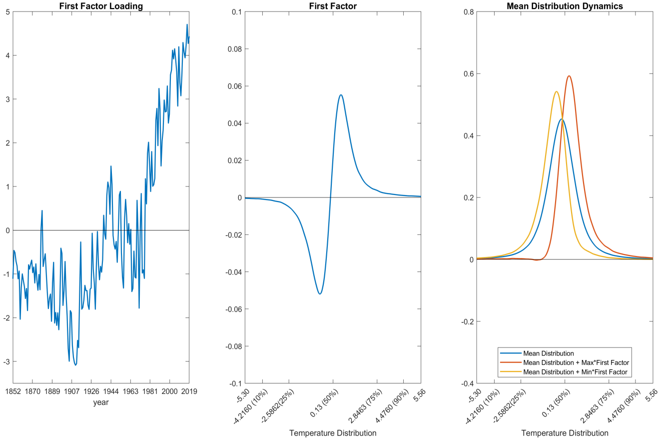


Cumulative Scree Plot



Three leading factors explain 98% of the variations in the time series of temperature distributions.

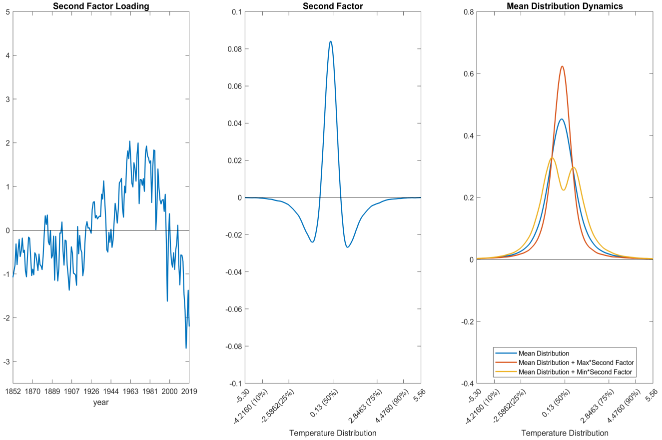
First Factor



1^{st} factor: $(+)$ on anomalies $>$ median, $(-)$ on $<$ median

$\Longrightarrow$ positive loading increases mean & skewness

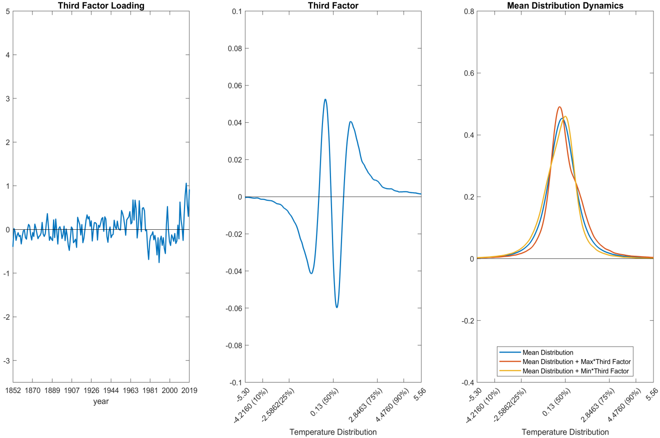
Second Factor



2^{nd} factor: $(+)$ on anomalies near mode, $(-)$ otherwise

$\Longrightarrow$ positive loading decreases variance, kurtosis

Third Factor



3^{rd} factor: positive loading appears to increase skewness

Aggregate Climate SVAR

We postulate that (z_t) evolves according to

$$A_0 z_t = \mu + \sum_{i=1}^{p} A_i z_{t-i} + \sum_{k=0}^{q} C_i \tilde{x}_{t-k} + e_t,$$

with 4×4 matrices $A_0, A_1, \ldots, A_p$ and 4×2 matrices $C_0, C_1, \ldots, C_q$, and where $(\tilde{x}_t)$ is a series of fitted residuals from fitting (x_t) to a VAR. (Lag orders chosen by BIC.)

Reduced form errors (ε_t) relate to structural errors (e_t) by

$$\varepsilon_t = A_0^{-1} e_t,$$

where A_0^{-1} corresponds to B_m above.

Note: μ and initial condition may "soak up" long-run relationships.

Identifiying an SVAR

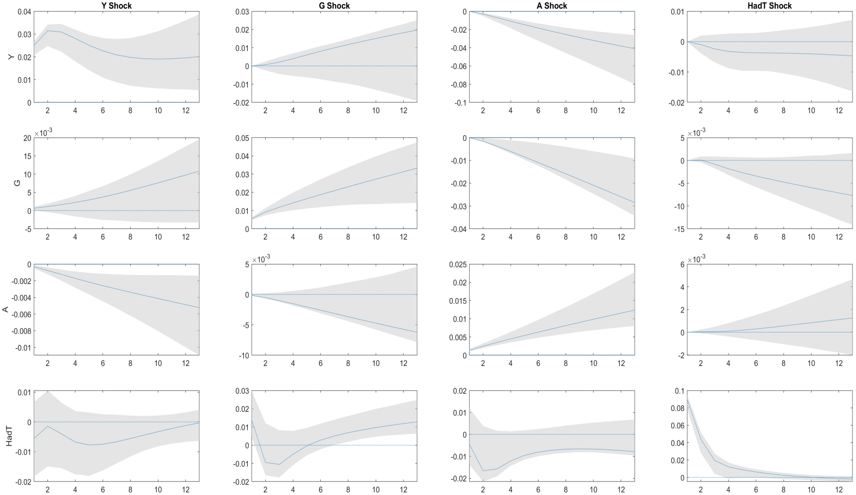
We postulate a structure on A_0^{-1} given by

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^Y \\ \varepsilon_t^G \\ \varepsilon_t^A \\ \varepsilon_t^T \end{bmatrix} = \begin{bmatrix} a_{YY} & 0 & 0 & 0 \\ a_{GY} & a_{GG} & 0 & 0 \\ a_{AY} & a_{AG} & a_{AA} & 0 \\ a_{TY} & a_{TG} & a_{TA} & a_{TT} \end{bmatrix} \begin{bmatrix} e_t^Y \\ e_t^G \\ e_t^A \\ e_t^T \end{bmatrix} = A_0^{-1} e_t$$

On impact, innovations in ...

- $a_{GY}, a_{AY} \neq 0$: ... production $\Rightarrow$ emissions
- $a_{TY} \neq 0$: ... production $\Rightarrow$ temperature
- $a_{YG}, a_{YA} = 0$: ... emissions $\not\Rightarrow$ production
- $a_{YT} = 0$: ... but temperature $\Rightarrow$ production! (not ideal)
- $a_{TG}, a_{TA} \neq 0$: ... emissions $\not\Rightarrow$ temperature
- $a_{GA} = 0$: ... emissions $\not\Rightarrow$ each other, but a_{AG} unrestricted
- $a_{GT}, a_{AT} = 0$: ... temperature $\not\Rightarrow$ emissions

Aggregate SVAR: Impulse Responses Analysis



90% intervals estimates shown here and henceforth.

Results: Conventional SVAR

Not surprisingly, all structural shocks impact their own series. The impact is permanent for Y, G, and A, but less so for T.

A few more insights...

- "beginning"
  - $e_t^Y \nearrow G$ (expected, but **not** significant)
  - $e_t^Y \searrow A$ (expected, significant)
- "end"
  - $e_t^T \searrow Y$ (expected, but **not** significant)
- "murky middle"
  - $e_t^A \searrow G, Y$ (**un**expected)
  - $e_t^G \nearrow T$ (expected, significant on impact, after 6 yrs)
  - $e_t^A \searrow T$ (**wrong** sign, **not** significant)

A Novel Functional Structural VAR Model

Two Global Exogenous Variables:

- S: Solar activity,
- V: Stratospheric aerosols from volcanic activity

Three Global Endogenous Variables:

- Y: Global economic production
- G: Greenhouse gas concentration
- A: Tropospheric aerosols and land use

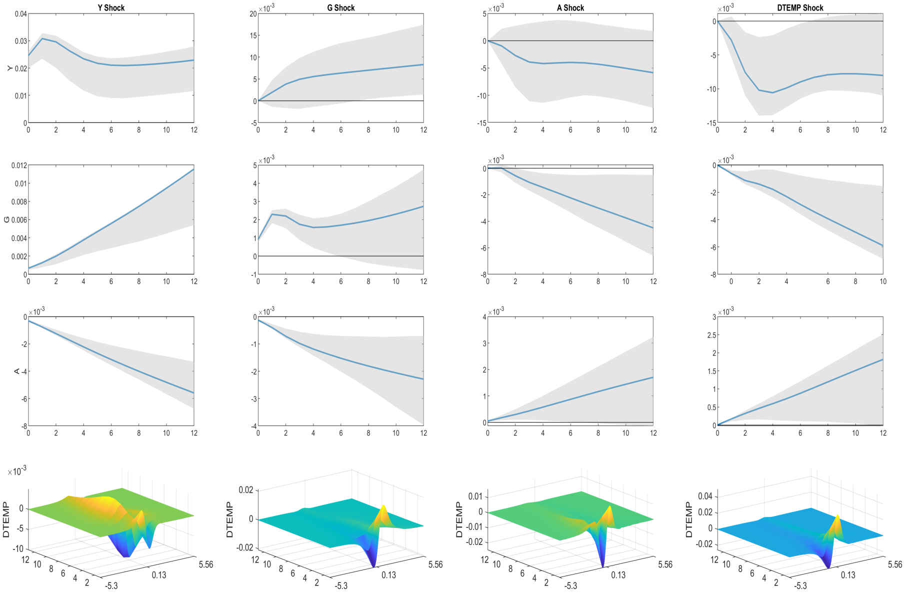
One Functional Variable:

- $FTemp$: Densities of temperature anomaly distribution

FSVAR: IRFs

- First: Impulse response function surfaces of the functional variable along with the impulse response functions (IRFs) of the three aggregate endogenous variables in response to the four structural shocks identified.
- Next: Slices of the IRF surfaces of the functional variables observed at 10 different horizons h from impact to twelve years later

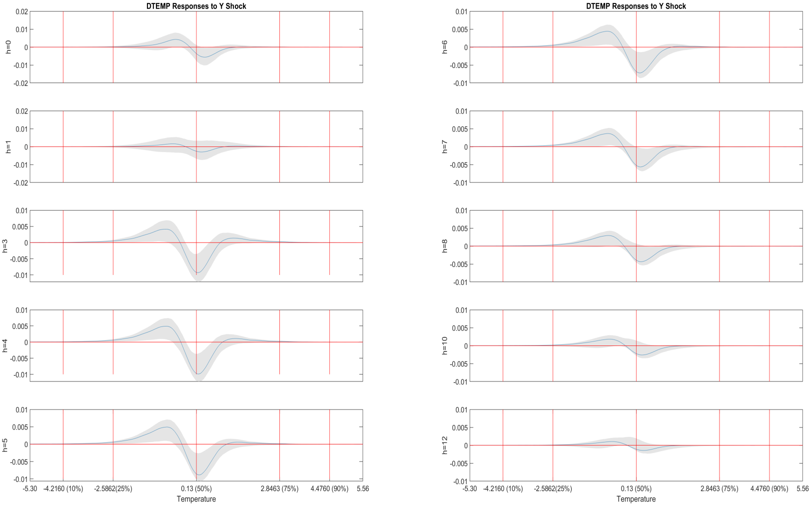
FSVAR: IRFs


Y Shock
G Shock
A Shock
DTEMP Shock
Y
G
A
DTEMP
DTEMP
DTEMP
DTEMP

FSVAR: Interpretations of the IRFs

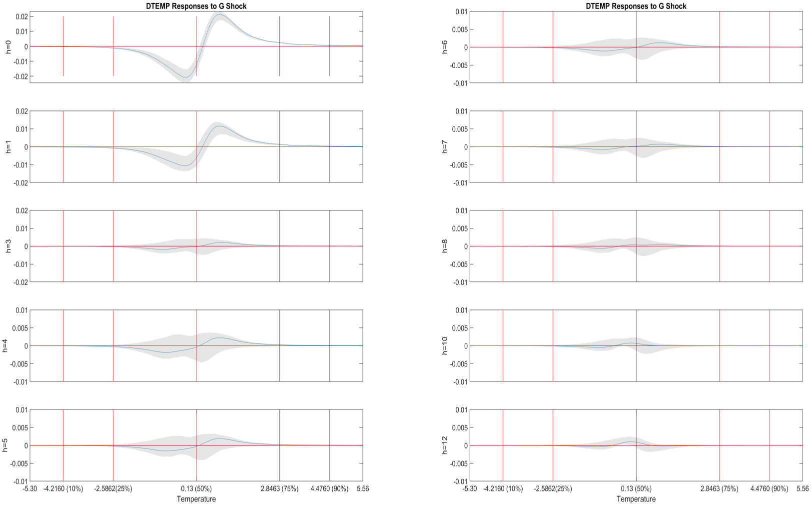
- "beginning"
  - $e_t^Y \nearrow G$ (expected, **now** significant)
  - $e_t^Y \searrow A$ (expected, **still** significant)
- "end"
  - $e_t^T \searrow Y$ (expected, **now** significant)
- "murky middle"
  - $e_t^A \searrow G$, $e_t^G \searrow A$, $e_t^G \nearrow Y$ (**un**expected)
    - Speculation: persistent measurement error
  - $e_t^G \nearrow T$, $e_t^A \nearrow T$ (significant? yes, next few slides)

Responses of Temperature Distribution to Y Shock



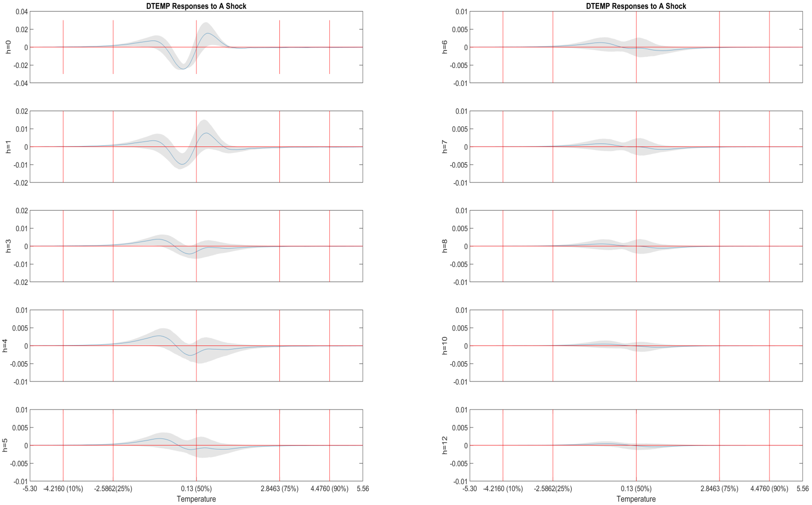
e_t^Y : $\searrow$ $mean$ on impact; $\nearrow$ $variance$, $\searrow$ $skewness$ at 3-8 years

Responses of Temperature Distribution to G Shock



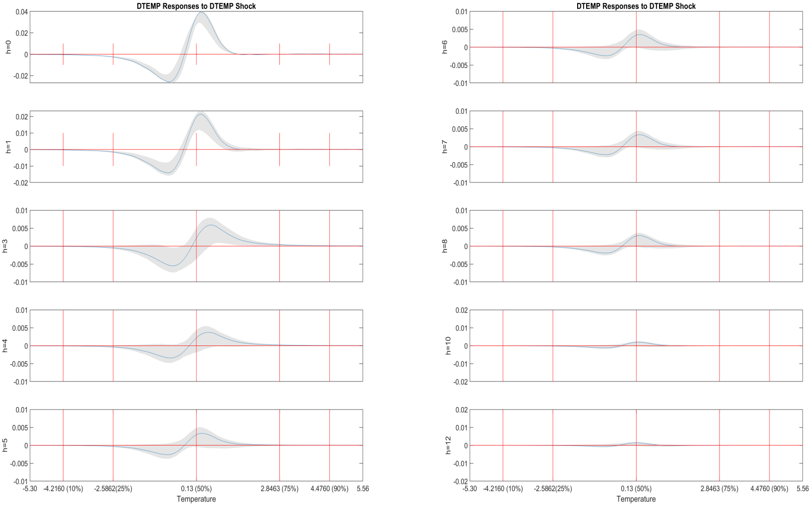
$e_t^G : \nearrow mean$ on impact and for 1-2 years

Responses of Temperature Distribution to A Shock



$e_t^A : \nearrow mean, \searrow skewness$ on impact and for a few years

Responses of Temperature Distribution to T Shock

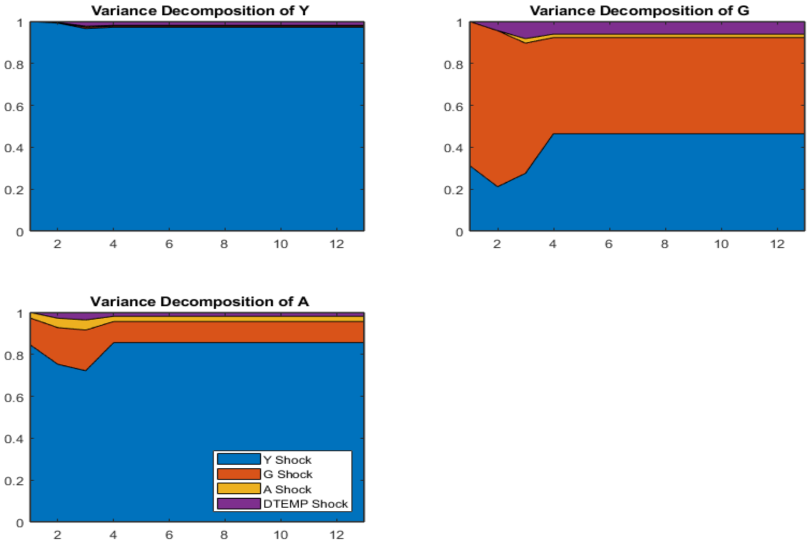


e_t^T : $\nearrow$ $mean$ on impact and for a few years

FSVAR: FEVDs

- First: Forecast error variance decomspoistion (FEVDs) of the three aggregate endogenous variables, Y, G, A
  - What explains unexpected movements in these series?
- Next: FEVDs of the functional variable at six key temperature anomalies:
  - -5.300, -4.216, -2.586, 0.130, 2.846, 4.476 (minimum; 10^{th}, 25^{th}, 50^{th}, 75^{th}, 90^{th} percentiles)

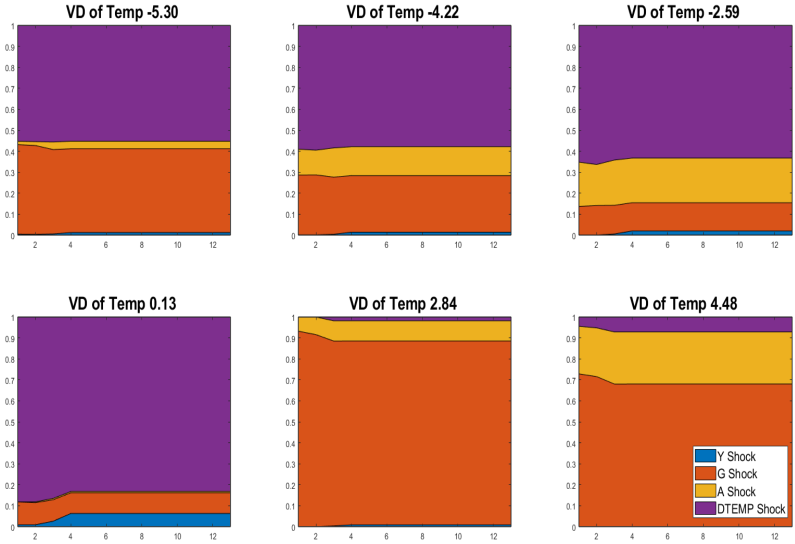
FEVD of Aggregate Variables, Y, G, and A



Results: FEVD of Aggregate Variables

- Shocks to Y account for much/most of the variation in forecast error of all three variables
- Shocks to A and T account for almost none
- Why do shocks to Yn account for more of the variation in forecast error of A than of G?
  - G is highly persistent in the atmosphere, while A is not
  - A relates to emissions, while G relates to cumulative emissions

FEVD of Functional Var $FTemp$ at Specific Values



Each panel presents FEVD of FTemp at a specific value indicated on top.

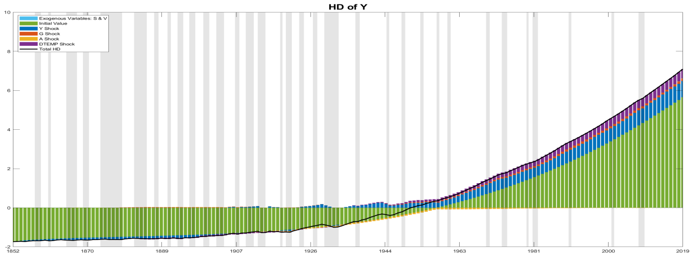
Results: FEVD of Functional Variable

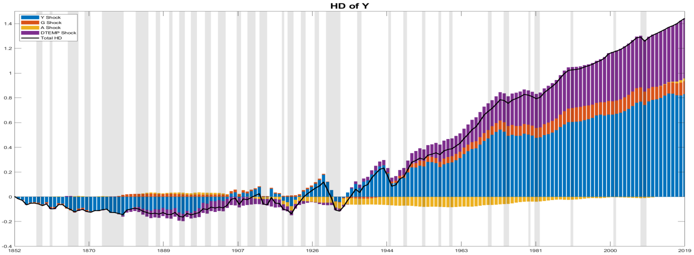
- Shocks to Y account for almost none of the variation in forecast error in the frequency of any temperature anomaly
- Shocks to A account for almost none or up to about 20%
- Shocks to G account for most of the variation in forecast error of frequency of anomalies above the median (0.13°C)
- Residuals shocks to T account for most of the variation in forecast error of frequency of anomalies at or below the median

FSVAR: Historical Decomposition

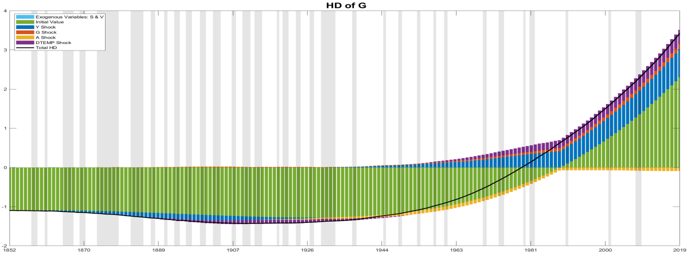
- First: Historical decompositions (HDs) of the three aggregate variables, Y, G, and A.
- Next: HD of the functional variable at three key temperature anomalies:
  - -4.216, 4.476, 0.130 (10^{th}, 90^{th}, 50^{th} percentiles)

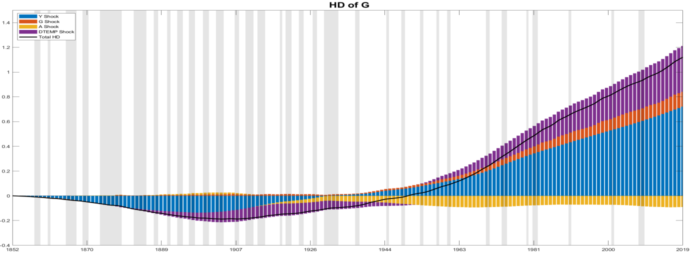
Historical Decomposition of Y



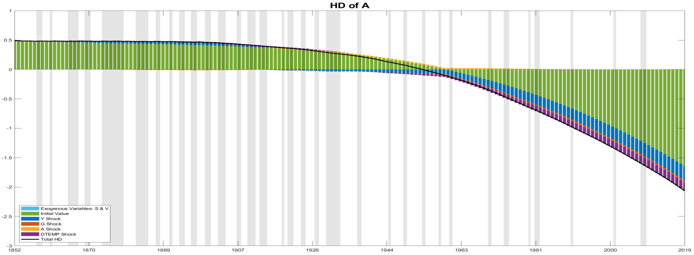


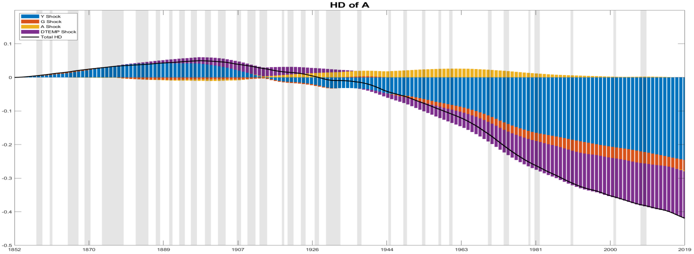
Historical Decomposition of G





Historical Decomposition of A

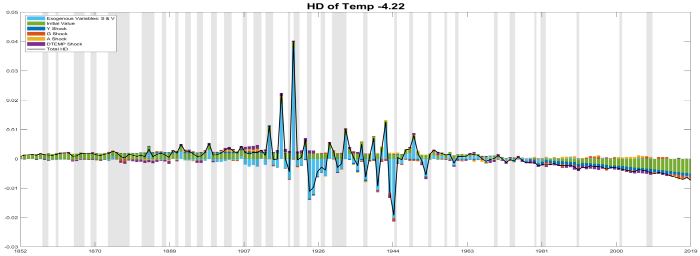


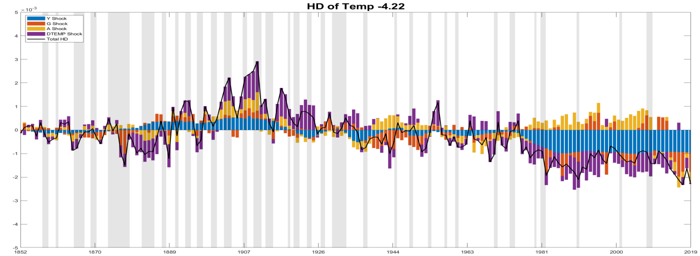


Results: HD of Aggregate Variables

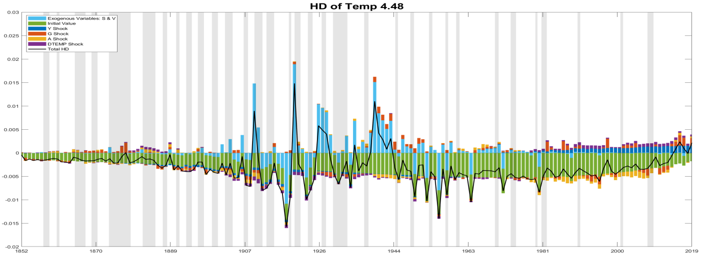
- Most salient in all three are the long-run effects, effects of initial conditions and conditional means
- Net of these, production shocks primarily drive all three aggregate variables (Y, G, A)
  - $\Longrightarrow$ Economic activity drives emissions
- Temperature shocks secondarily drive all three variables
  - $\Longrightarrow$ Evidence of a short-run negative feedback loop

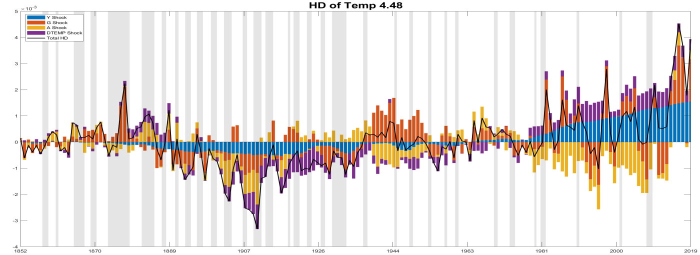
Historical Decomposition of $FTemp$: 10^{th} Percentile



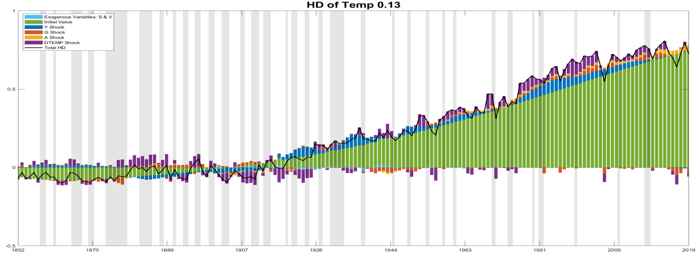


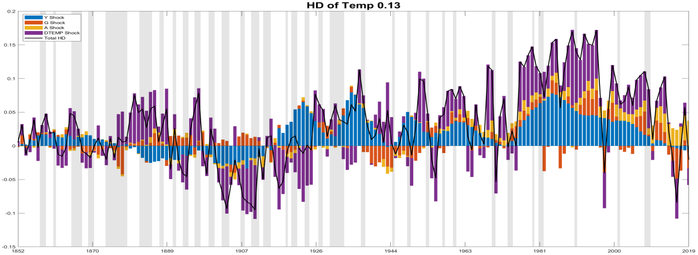
Historical Decomposition of $FTemp$: 90^{th} Percentile





Historical Decomposition of $FTemp$: Median





Results: HD of Functional Variables

- 10^{th} and 90^{th} percentiles mainly driven by exogenous shocks; relatively small long-run movement
- Median anomaly (near zero) shows a steady long-run increase
- All anomalies show substantial fluctuations from shocks to Y:
  - decreasing density below the median anomaly
  - increasing density above the median anomaly
- Residual (temperature shocks) also important in short-run

Some Takeaways

- We introduce a functional SVAR that is a mixture of a traditional VAR and pure functional VAR with identified structural errors.
- Estimation is accomplished using functional principal components which then allow tradition VAR techniques.
- Applying the FSVAR to a model of the climate system shows
  - effects of shocks to economic activity on climate forcings from greenhouse gases and tropospheric aerosols,
  - effects of shocks to these series on temperature distributions, primarily increasing mean, possibly decreasing skewness, and
  - effects of shocks in temperature decrease economic activity.