A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET

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GLOBAL CARBON CYCLE

Friedlingsstein et al. (2019), The global carbon budget 2019, Earth System Science Data 11(4), 1783-1838

www.globalcarbonproject.org
OUTLINE OF THE TALK

1. Models for the components of the global carbon budget
2. The dynamics of atmospheric concentrations C
3. The system model
4. Estimation: Residual diagnostics, Residual processes, Parameter estimates
5. Simulation
6. Discussion: Budget imbalance, airborne fraction, sink rate
7. Nowcasts and forecasts
8. Projections: Long-term scenarios until 2100
9. Conclusions
ANTHROPOGENIC EMISSIONS

\[ \Delta E_t = 0.12 \pm 0.02 + \eta_{5,t} \]
\[ \eta_{5,t} \sim N(0, 0.026) \]

\[ \Delta E_t = 3.15 \pm 0.30 \Delta \log GDP_{t \text{world}} - 0.11 \pm 0.08 I_{1973} - 0.18 \pm 0.08 I_{1980} - 0.25 \pm 0.08 I_{1991} - 0.65 \pm 0.18 I_{1997} + \eta_{5,t} \]
\[ \eta_{5,t} \sim N(0, 0.006) \]

\[ \Delta \log GDP_{t \text{world}} \approx 0.034 \]
SINKS LINEAR IN CONCENTRATIONS

\[ S_{LND_t} = \beta \log \left( \frac{C_t}{C_0} \right) \]

\( C_0 \) pre-industrial concentration 593GtC or 279ppm

\[ S_{LND_t} = \frac{a(C_t - C_b)}{1 + b(C_t - C_b)} \]

\( C_b = 80\text{GtC NPP−zerolevel}, a, b > 0 \)

\[ S_{OCN_t} = k_o(pCO2_t^a - pCO2_t^b) \]

Joos et al. (1996, 2001)
Meinshausen et al. (2011)
LAND SINK

\[ S_{LN}D_t = \frac{7.23}{0.88} \frac{C_t}{C_0} \]

\[ S_{LN}D_t = \frac{7.20}{0.90} \frac{C_t}{C_0} + 0.57 \times 0.12 SOI_t \]

“Moisture sensitivities of both productivity and decomposition are important for capturing the response of the net flux to such [La Nina] events.” Haverd et al. (2018, p. 3013)
When the winds are strongest during the cold cycle of ENSO deep upwelling occurs and [ocean CO2 partial pressure] values are at a maximum. *Feely et al. (1999, p. 599)*

\[
S_{OCN_t} = \frac{5.53}{(0.51)} \frac{C_t}{C_0} - 0.05 \frac{C_t}{C_0} - 0.01 S_{OI_t}
\]
\[ \Delta C = \text{GROWTH IN ATM. CONCENTRATIONS} \]

\[ \Delta C_t = E_t - S_{\text{LND}}_t - S_{\text{OCN}}_t \]
THE DYNAMICS OF C

\[ \Delta C_t = E_t - S_{LND_t} - S_{OCN_t} \]
\[ = E_t - \beta_1^* C_t - \beta_2^* C_t + \epsilon_t, \quad \epsilon_t \sim I(0) \]
\[ (1 + \beta_1^* + \beta_2^*) C_t - C_{t-1} = E_t + dt + x_t + \epsilon_t \]
\[ (1 - qL) C_t = qE_0 + qdt + qx_t + q\epsilon_t \]

Three insights:
\[ C_t = q^t \left[ C_0 - \frac{qE_0}{1 - q} + \frac{dq^2}{2(1 - q)^2} \right] + \left[ \frac{qE_0}{1 - q} - \frac{dq^2}{2(1 - q)^2} \right] + \frac{dq}{1 - q} t + \sum_{j=0}^{t-1} q^{j+1} x_{t-j} + \sum_{j=0}^{t-1} q^{j+1} \epsilon_{t-j} \]
\[ = o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1) \]

Thus,
\[ \Delta C_t = I(0) \]

But,
\[ (1 - qL)(1 - L) C_t = qd + qdx_t + q\Delta \epsilon_t = I(0) \]

\[ \beta_t^* = \frac{\beta_t}{C_0} \approx 0.01 \]
\[ x_t = \sum_{i=1}^{t} \eta_{5,i} \]
\[ q := \frac{1}{1 + \beta_1^* + \beta_2^*} \approx \frac{1}{1.02} \]
THE SYSTEM MODEL

**State equation Model 1**

\[
S_{LN}t_{t+1}^* = \frac{\beta_1}{C_0} C_{t+1}^*
\]

\[
S_{OCN}t_{t+1}^* = \frac{\beta_2}{C_0} C_{t+1}^*
\]

\[
E_{t+1}^* = E_t^* + d + \eta_{5,t}
\]

\[
C_{t+1}^* = C_t^* + G_{ATM}t_{t+1}
\]

\[
G_{ATM}t_{t+1}^* = E_{t+1}^* - S_{LN}t_{t+1}^* - S_{OCN}t_{t+1}^*
\]

**State equation Model 2**

\[
S_{LN}t_{t+1}^* = \frac{\beta_1}{C_0} C_{t+1}^* + \beta_3 S{O}l_{t+1}
\]

\[
S_{OCN}t_{t+1}^* = \frac{\beta_2}{C_0} C_{t+1}^* + \beta_4 S{O}l_{t+1}
\]

\[
E_{t+1}^* = E_t^* + \beta_5 \Delta \log GDP_{t+1} + \text{dummies} + \eta_{5,t}
\]

**Measurement equation**

\[
C_t = C_t^* + X_{1,t}
\]

\[
S_{LN}t_{t} = S_{LN}t_{t+1}^* + X_{2,t}
\]

\[
S_{OCN}_t = S_{OCN}t_{t+1}^* + X_{3,t}
\]

\[
E_t = E_{t-1}^* + X_{4,t}
\]
# RESIDUAL DIAGNOSTICS

## Model 1

<table>
<thead>
<tr>
<th>Residual</th>
<th>mean</th>
<th>std dev</th>
<th>skew</th>
<th>kurt</th>
<th>LB(1)</th>
<th>JB</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.030</td>
<td>0.953</td>
<td>0.313</td>
<td>3.061</td>
<td>1.671</td>
<td>0.955</td>
<td>1.659</td>
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<tr>
<td>E</td>
<td>0.204</td>
<td>0.988</td>
<td>-1.372</td>
<td>8.084</td>
<td>0.002</td>
<td>80.66***</td>
<td>1.897</td>
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<tr>
<td>S_LND</td>
<td>-0.152</td>
<td>0.985</td>
<td>0.033</td>
<td>2.960</td>
<td>0.202</td>
<td>0.014</td>
<td>2.064</td>
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<tr>
<td>S_OCN</td>
<td>0.051</td>
<td>0.997</td>
<td>0.263</td>
<td>2.843</td>
<td>0.050</td>
<td>0.729</td>
<td>1.906</td>
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## Model 2

<table>
<thead>
<tr>
<th>Residual</th>
<th>mean</th>
<th>std dev</th>
<th>skew</th>
<th>kurt</th>
<th>LB(1)</th>
<th>JB</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.031</td>
<td>0.879</td>
<td>-0.239</td>
<td>3.666</td>
<td>0.212</td>
<td>1.569</td>
<td>1.836</td>
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<tr>
<td>E</td>
<td>0.592</td>
<td>0.769</td>
<td>0.351</td>
<td>3.147</td>
<td>1.880</td>
<td>1.198</td>
<td>1.636</td>
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<tr>
<td>S_LND</td>
<td>-0.074</td>
<td>0.983</td>
<td>0.042</td>
<td>2.343</td>
<td>0.478</td>
<td>1.023</td>
<td>2.172</td>
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<tr>
<td>S_OCN</td>
<td>0.032</td>
<td>0.961</td>
<td>0.135</td>
<td>3.441</td>
<td>0.242</td>
<td>0.622</td>
<td>2.093</td>
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RESIDUAL PROCESSES

\[ X_{1,t} \text{ in } C_t \]
\[ X_{2,t} \text{ in } S_{LND_t} \]
\[ X_{3,t} \text{ in } S_{OCN_t} \]
\[ X_{4,t} \text{ in } E_t \]
## PARAMETER ESTIMATES

<table>
<thead>
<tr>
<th>Model 2</th>
<th>Coefficients</th>
<th>Variances</th>
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</thead>
<tbody>
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<td></td>
<td>Coefficients</td>
<td>estimate</td>
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<tr>
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<td>$c_1$ (filt.)</td>
<td>-6.77</td>
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<tr>
<td></td>
<td>$c_2$ (filt.)</td>
<td>-5.35</td>
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<tr>
<td></td>
<td>$\beta_1$</td>
<td>7.20</td>
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<tr>
<td></td>
<td>$\beta_2$</td>
<td>5.57</td>
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<tr>
<td></td>
<td>$\beta_3$ (filt.)</td>
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<tr>
<td></td>
<td>$\beta_4$ (filt.)</td>
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<td></td>
<td>$\beta_5$ (filt.)</td>
<td>3.15</td>
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<td></td>
<td>$\beta_6$ (filt.)</td>
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<td>$\beta_7$ (filt.)</td>
<td>-0.18</td>
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<tr>
<td></td>
<td>$\beta_8$ (filt.)</td>
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</tr>
<tr>
<td></td>
<td>$\beta_9$ (filt.)</td>
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<tr>
<td></td>
<td>$\phi_1$</td>
<td>0.86</td>
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<td></td>
<td>$\phi_3$</td>
<td>0.74</td>
</tr>
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</table>
SIMULATIONS OF MODEL 1

σ_4^2 exhibits “pile-up” problem (Stock and Watson 1998)
BUDGET IMBALANCE

(A) Smoothed
\(-\Delta X_1 + X_2 + X_3\)

(B) One-year ahead predictions

(C) Components
\(-\Delta X_1, -X_2, -X_3\)
AIRBORNE FRACTION AND SINK RATE

Airborne Fraction

Sink Rate

\[ AF = \frac{\Delta C}{E} \]

\[ SR = \frac{S_{LND} + S_{OCN}}{C} \]
Forecasts of World GDP growth from IMF and World Bank

<table>
<thead>
<tr>
<th></th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
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</thead>
<tbody>
<tr>
<td>IMF</td>
<td>3.2%</td>
<td>-4.9%</td>
<td>5.4%</td>
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<tr>
<td>World Bank</td>
<td>2.6%</td>
<td>-5.2%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Data</td>
<td>2.4%</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Forecasts of SOI from forecast model of monthly SOI data 1866-1920, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years.
PROJECTIONS TO 2100

Scenarios:
- 1% GDP growth
- 3.4% GDP growth
- 5% GDP growth
- Piketty scenario:
  - 3.4% til 2030
  - 3% til 2050
  - 1.5% til 2100
CONCLUSIONS

• Specification of state-space model for Global Carbon Budget
• World GDP as driver in emissions
• Sinks: linear in CO2 concentrations and in SOI
• CO2 concentrations are I(1) ranging on I(2)
• Model allows for forecasting, projections, study of key variables such as airborne fraction and sink rate

Future directions
• Include ensemble members for S_LND and S_OCN
• Factor model for drift in emissions using large macroeconomic dataset
• Higher resolution on Global Carbon Cycle module (MAGICC)
• Connection to temperatures (Energy Balance Models)
• Cointegration analysis