A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET

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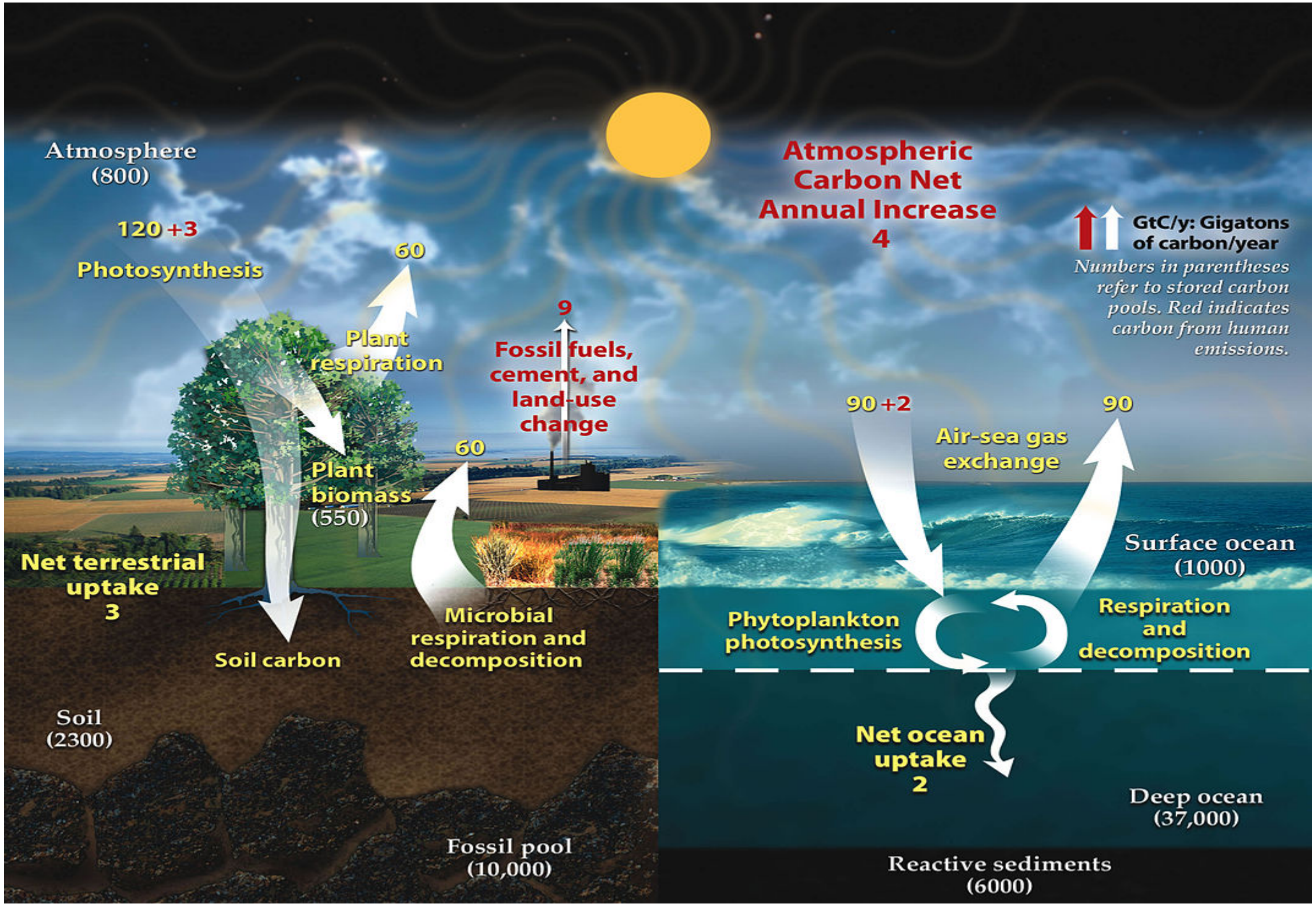
CLIMATE ECONOMETRICS VIRTUAL SEMINAR SERIES
15. DECEMBER 2020

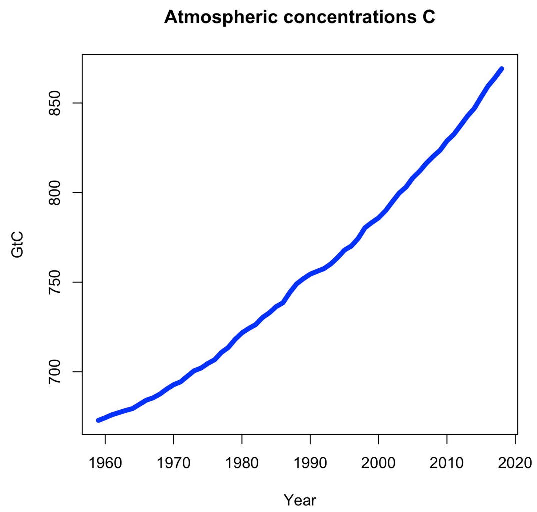
GLOBAL CARBON CYCLE



Source:
Wikipedia

DATA

GLOBAL CARBON PROJECT

Δ


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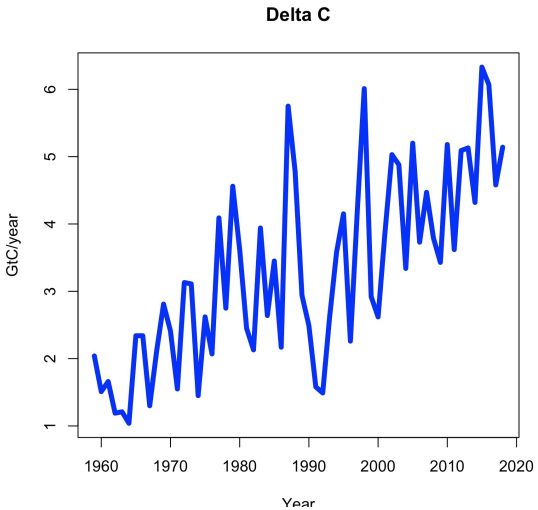

www.globalcarbonproject.org

Friedlingsstein et al. (2019),
The gobal carbon budget 2019,
Earth System Science Data 11(4),
1783-1838

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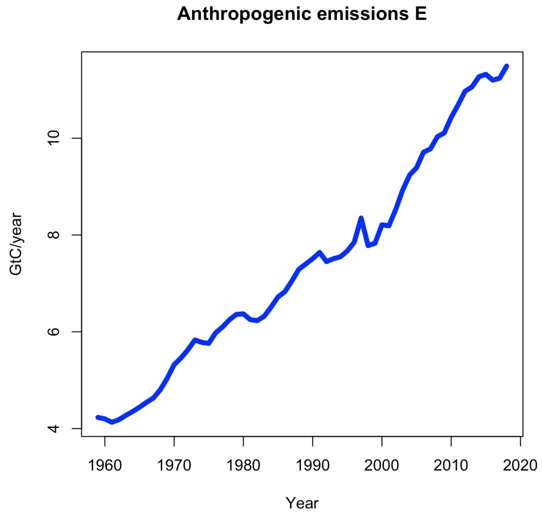

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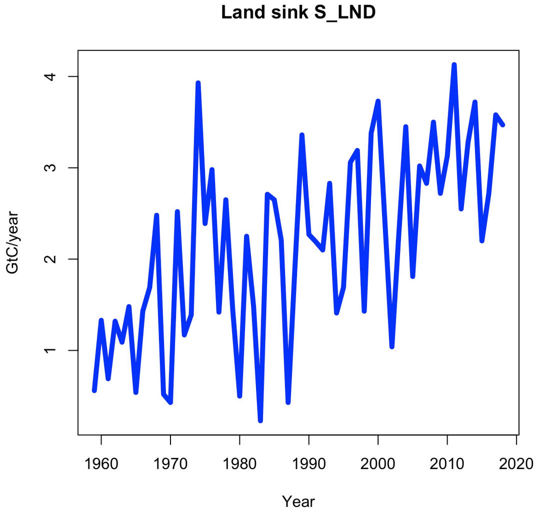

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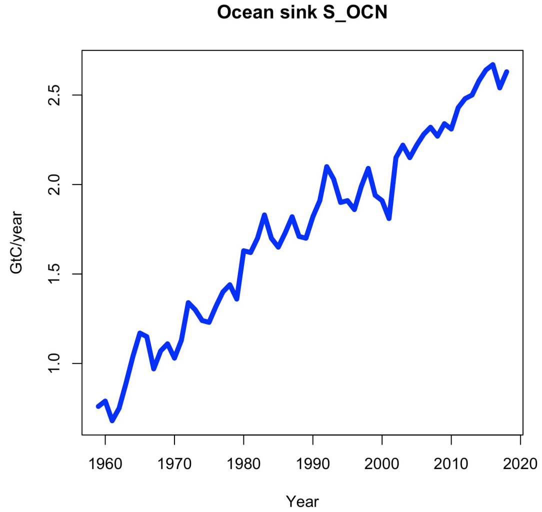

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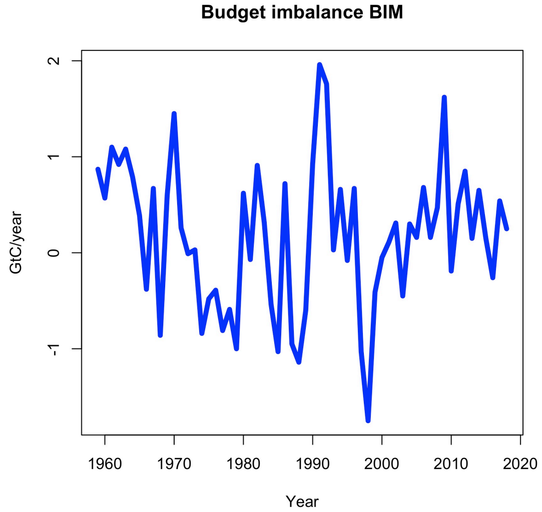
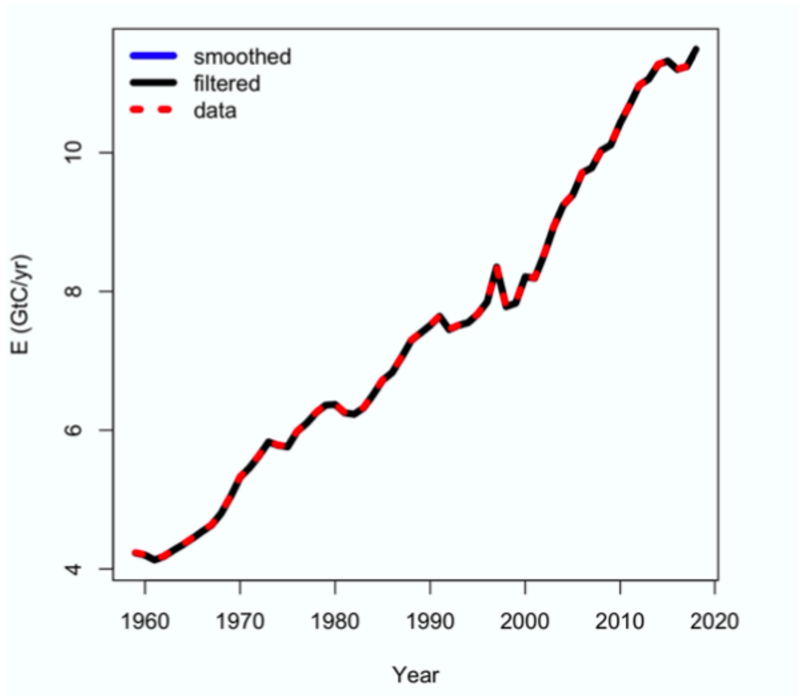
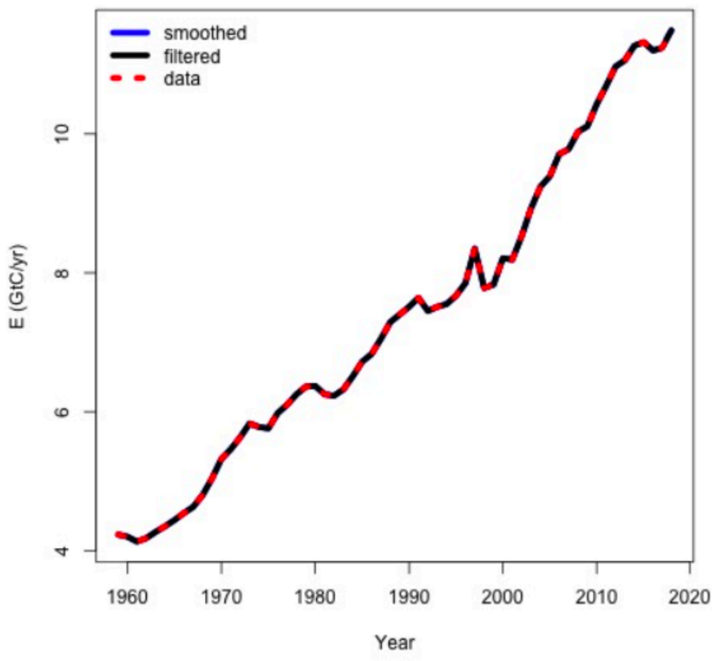


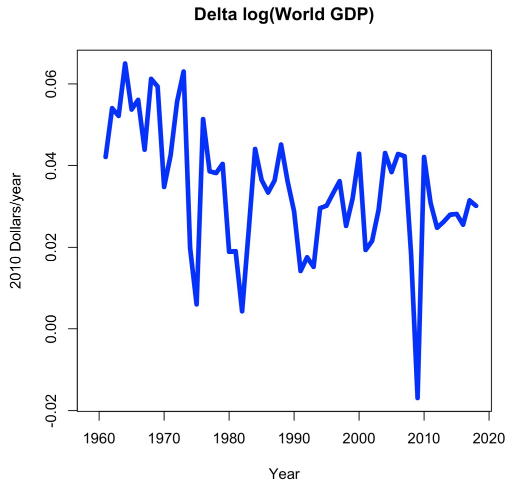
OUTLINE OF THE TALK

1. Models for the components of the global carbon budget
2. The dynamics of atmospheric concentrations C
3. The system model
4. Esimation: Residual diagnostics, Residual processes, Parameter estimates
5. Simulation
6. Discussion: Budget imbalance, airborne fraction, sink rate
7. Nowcasts and forecasts
8. Projections: Long-term scenarios until 2100
9. Conclusions

ANTHROPOGENIC EMISSIONS







$$\overline{\Delta \log GDP_t^{world}} \approx 0.034$$

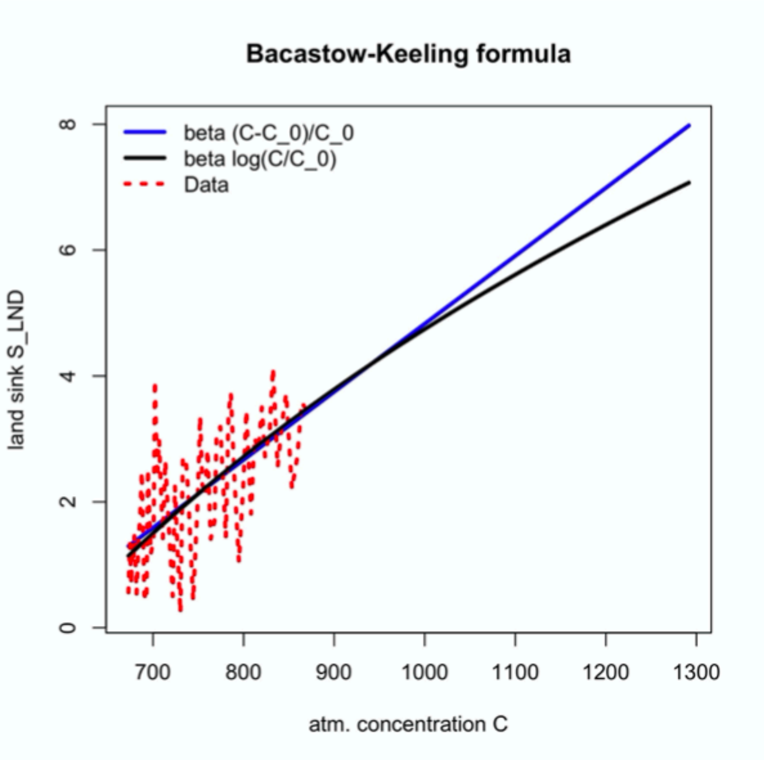
$$\Delta E_t = \underset{(0.02)}{0.12} + \eta_{5,t},$$

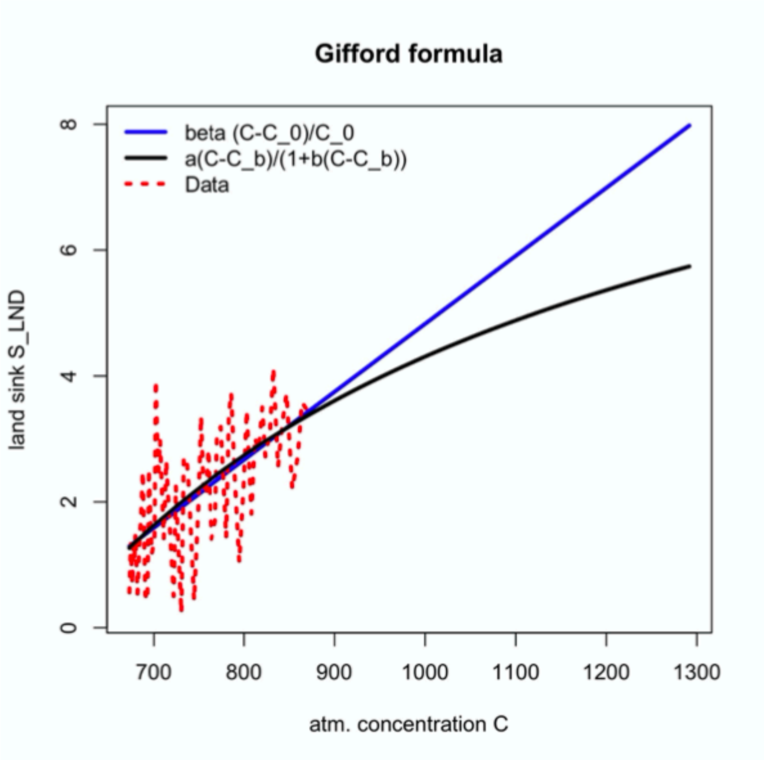
$$\eta_{5,t} \sim N(0, \underset{(0.004)}{0.026})$$

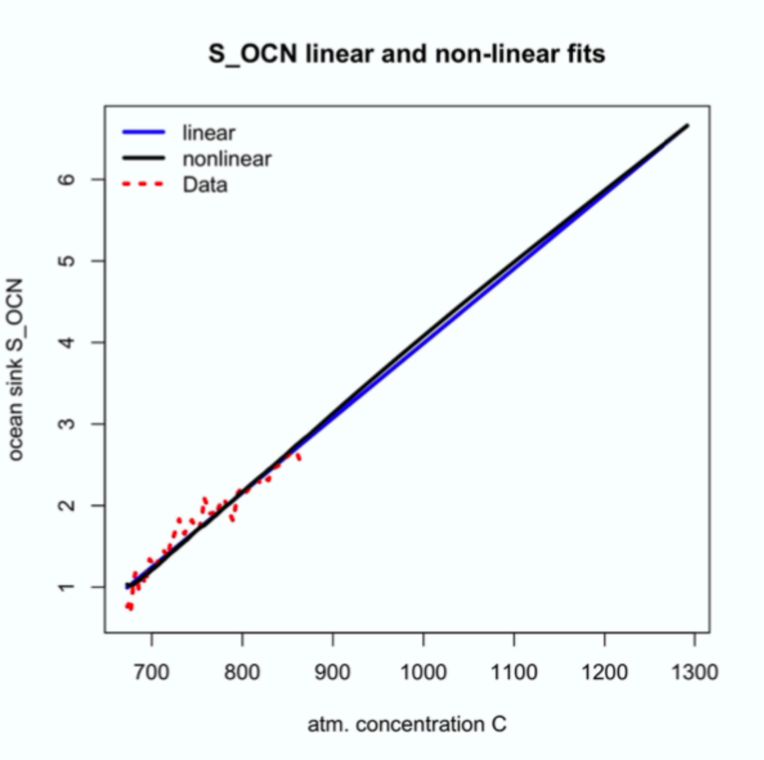
$$\Delta E_t = \underset{(0.30)}{3.15} \Delta \log GDP_t^{world} + \underset{(0.08)}{-0.11} I_{1973} + \underset{(0.08)}{-0.18} I_{1980} + \underset{(0.08)}{-0.25} I_{1991} + \underset{(0.18)}{-0.65} I_{1997} + \eta_{5,t}$$

$$\eta_{5,t} \sim N(0, \underset{(0.002)}{0.006})$$

SINKS LINEAR IN CONCENTRATIONS







$$S\_LND_t = \beta \log\left(\frac{C_t}{C_0}\right)$$

C_0 pre−industrial concentration 593GtC or 279ppm

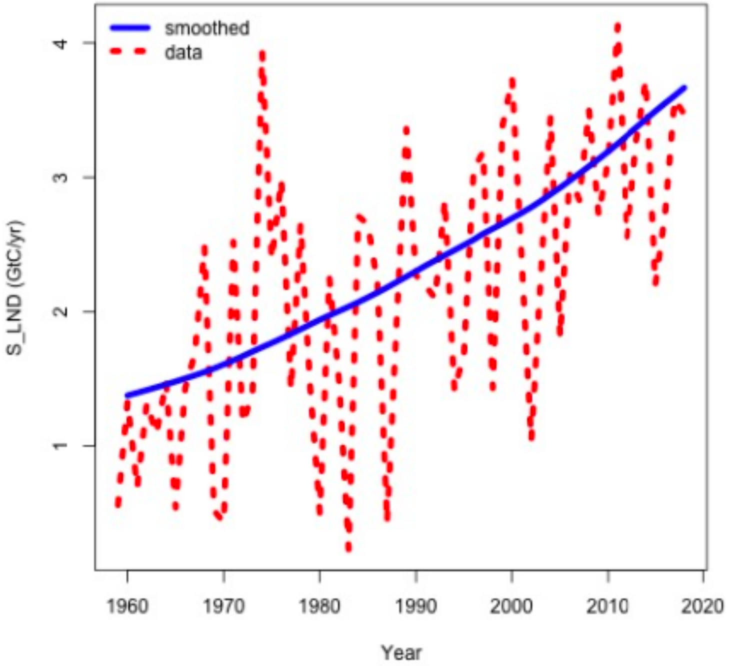
$$S\_LND_t = \frac{a(C_t - C_b)}{1 + b(C_t - C_b)}$$

$C_b = 80$GtC NPP−zerolevel, $a, b > 0$

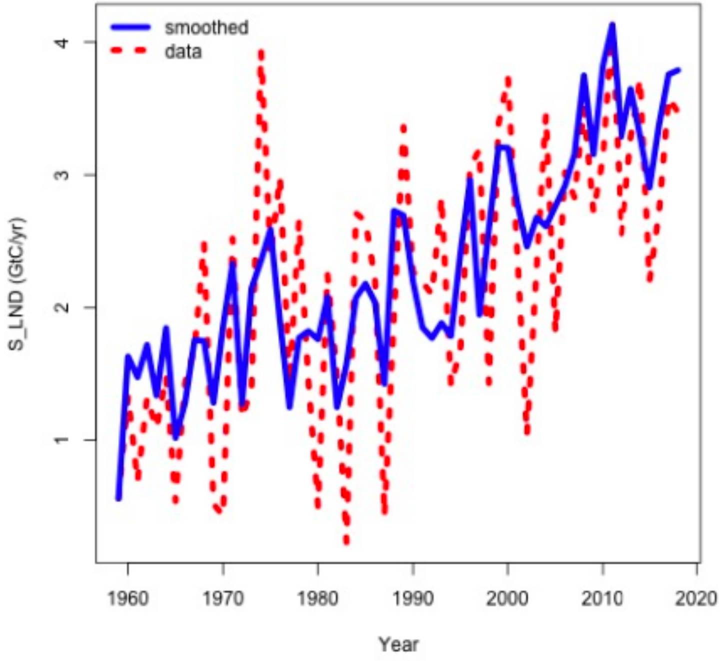
$$S\_OCN_t = k_O(pCO2_t^a - pCO2_t^s)$$

Joos et al. (1996, 2001)
Meinshausen et al. (2011)

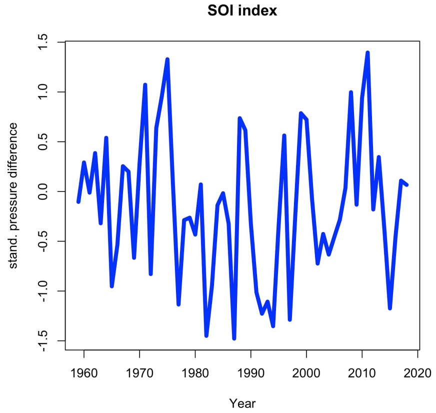
LAND SINK



$$S\_LND_t = \underset{(0.88)}{7.23}\frac{C_t}{C_0}$$

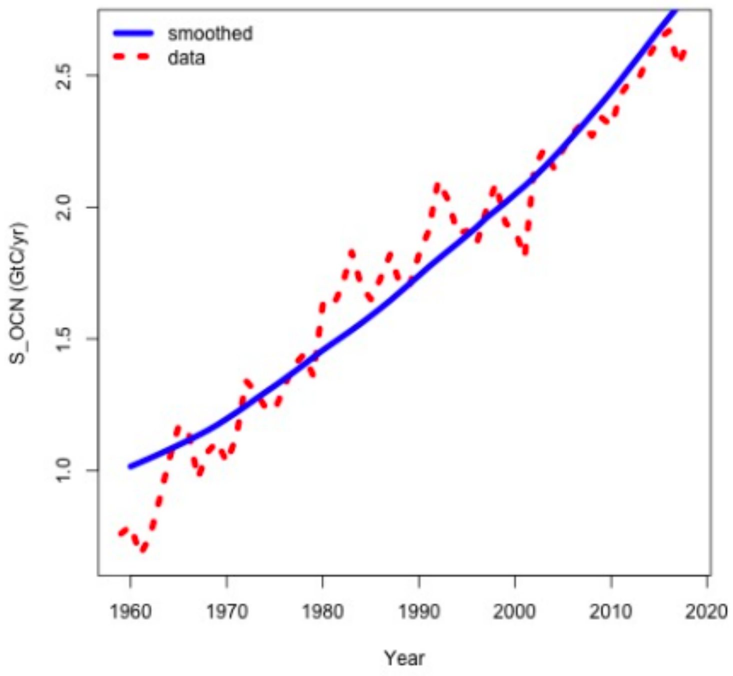


$$S\_LND_t = \underset{(0.90)}{7.20}\frac{C_t}{C_0} + \underset{(0.12)}{0.57}SOI_t$$

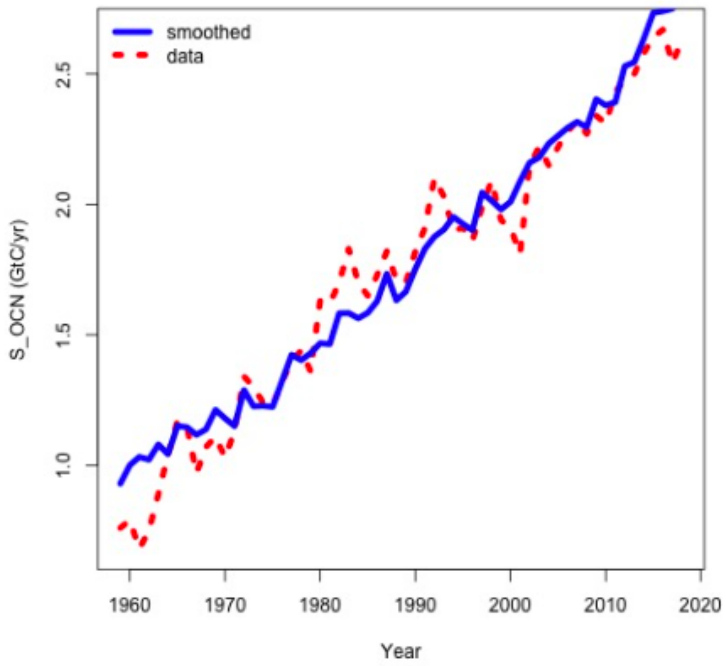


"Moisture sensitivities of both productivity and decomposition are important for capturing the response of the net flux to such [La Nina] events." Haverd et al. (2018, p. 3013)

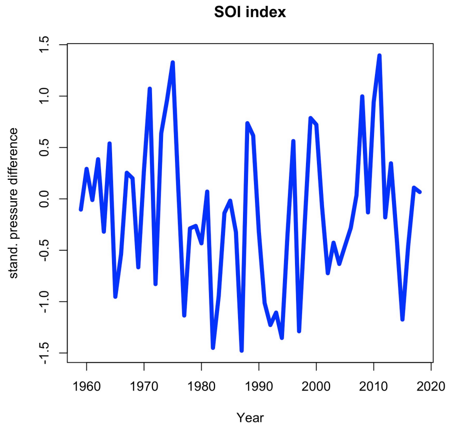
OCEAN SINK



$$S\_OCN_t = \underset{(0.51)}{5.53}\frac{C_t}{C_0}$$

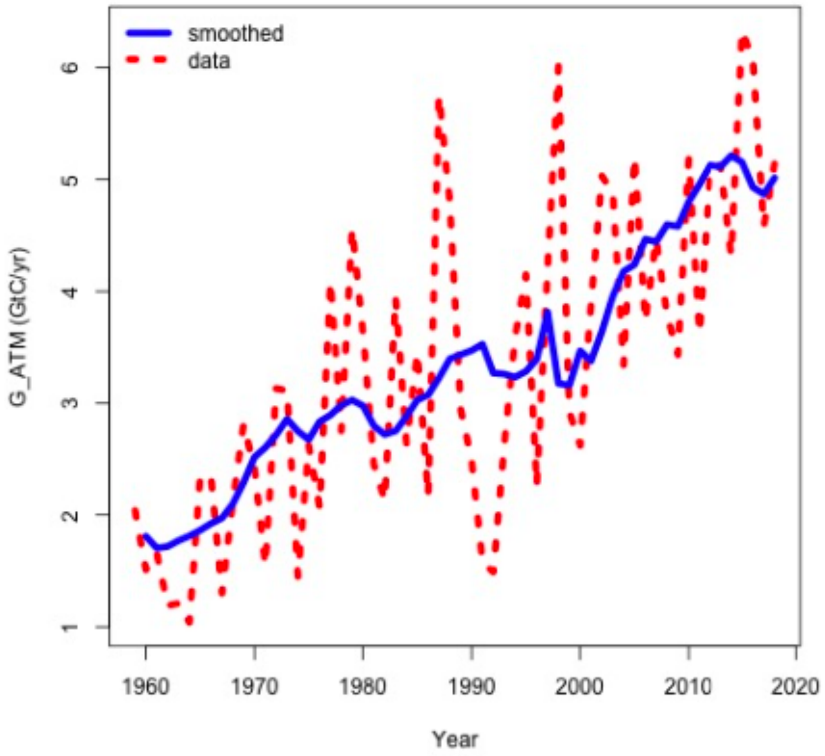


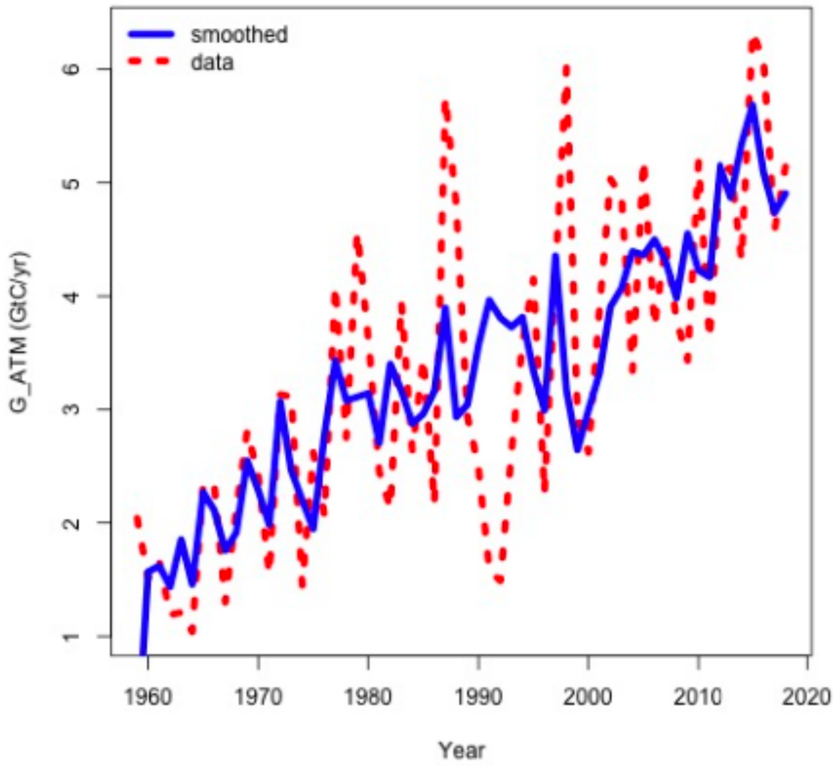
$$S\_OCN_t = \underset{(0.48)}{5.57}\frac{C_t}{C_0} - \underset{(0.01)}{0.05} SOI_t$$



"When the winds are strongest during the cold cycle of ENSO deep upwelling occurs and [ocean CO2 partial pressure] values are at a maximum ." Feely et al. (1999, p. 599)

ΔC = GROWTH IN ATM. CONCENTRATIONS





$$\Delta C_t = E_t - S\_LND_t - S\_OCN_t$$

THE DYNAMICS OF C

$$\Delta C_t = E_t - S\_LND_t - S\_OCN_t$$

$$= E_t - \beta_1^* C_t - \beta_2^* C_t + \varepsilon_t, \quad \varepsilon_t \sim I(0)$$

$$(1 + \beta_1^* + \beta_2^*)C_t - C_{t-1} = E_0 + dt + x_t + \varepsilon_t$$

$$(1 - qL)C_t = qE_0 + qdt + qx_t + q\varepsilon_t$$

$$\beta_i^* = \frac{\beta_i}{C_0} \approx 0.01$$

$$x_t = \sum_{i=1}^{t} \eta_{5,i}$$

$$q := \frac{1}{1 + \beta_1^* + \beta_2^*} \approx \frac{1}{1.02}$$

Three insights:

$$C_t = q^t \left[C_0 - \frac{qE_0}{1-q} + \frac{dq^2}{(1-q)^2}\right] + \left[\frac{qE_0}{1-q} - \frac{dq^2}{(1-q)^2}\right] + \frac{dq}{1-q}t + \sum_{j=0}^{t-1} q^{j+1} x_{t-j} + \sum_{j=0}^{t-1} q^{j+1} \varepsilon_{t-j}$$

$$= o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1)$$

Thus,

$$\Delta C_t = I(0)$$

But,

$$(1 - qL)(1 - L)C_t = qd + q\Delta x_t + q\Delta\varepsilon_t = I(0)$$

THE SYSTEM MODEL

State equation **Model 1**

$$S\_LND^*_{t+1} = \frac{\beta_1}{C_0} C^*_{t+1}$$

$$S\_OCN^*_{t+1} = \frac{\beta_2}{C_0} C^*_{t+1}$$

$$E^*_{t+1} = E^*_t + d + \eta_{5,t}$$

State equation **Model 2**

$$S\_LND^*_{t+1} = \frac{\beta_1}{C_0} C^*_{t+1} + \beta_3 SOI_{t+1}$$

$$S\_OCN^*_{t+1} = \frac{\beta_2}{C_0} C^*_{t+1} + \beta_4 SOI_{t+1}$$

$$E^*_{t+1} = E^*_t + \beta_5 \Delta \log GDP^{World}_{t+1} + \text{ dummies} + \eta_{5,t}$$

Measurement equation

$$C_t = C^*_t + X_{1,t}$$

$$S\_LND_t = S\_LND^*_t + X_{2,t}$$

$$S\_OCN_t = S\_OCN^*_t + X_{3,t}$$

$$E_t = E^*_{t-1} + X_{4,t}$$

$$C^*_{t+1} = C^*_t + G\_ATM^*_{t+1}$$

$$G\_ATM^*_{t+1} = E^*_{t+1} - S\_LND^*_{t+1} - S\_OCN^*_{t+1}$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X_{4,t} = \eta_{4,t}$$

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \\ \eta_{5,t} \end{bmatrix} \sim N\left[0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix}\right]$$

$$\eta_{4,t} \sim N(0, \sigma_4^2\, s_E^2\, I_{1996})$$

RESIDUAL DIAGNOSTICS

Model 1

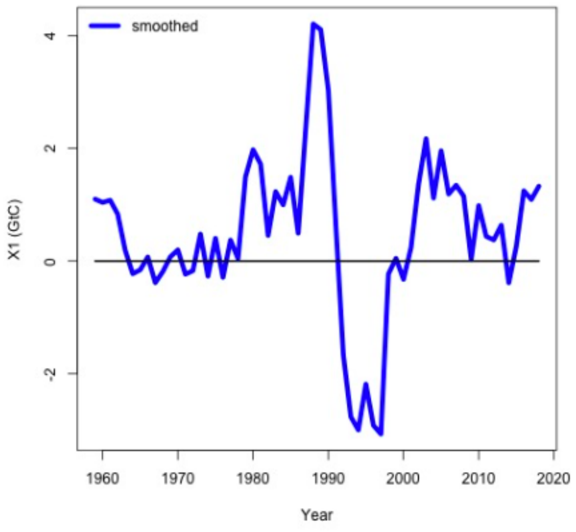
Residual	mean	std dev	skew	kurt	LB(1)	JB	DW
C	0.030	0.953	0.313	3.061	1.671	0.955	1.659
E	0.204	0.988	-1.372	8.084	0.002	80.66***	1.897
S_LND	-0.152	0.985	0.033	2.960	0.202	0.014	2.064
S_OCN	0.051	0.997	0.263	2.843	0.050	0.729	1.906

Model 2

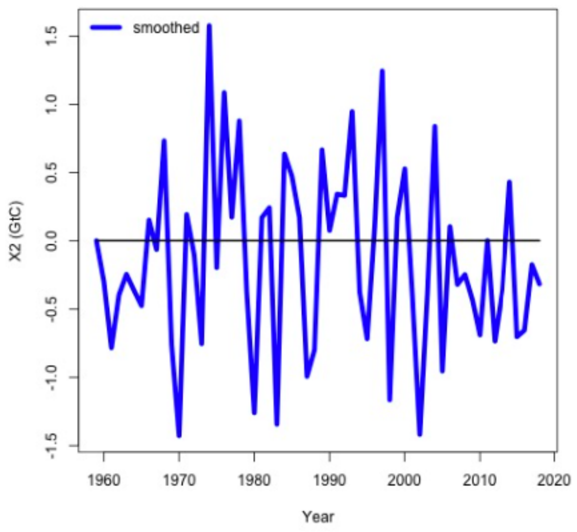
Residual	mean	std dev	skew	kurt	LB(1)	JB	DW
C	-0.031	0.879	-0.239	3.666	0.212	1.569	1.836
E	0.592	0.769	0.351	3.147	1.880	1.198	1.636
S_LND	-0.074	0.983	0.042	2.343	0.478	1.023	2.172
S_OCN	0.032	0.961	0.135	3.441	0.242	0.622	2.093

RESIDUAL PROCESSES

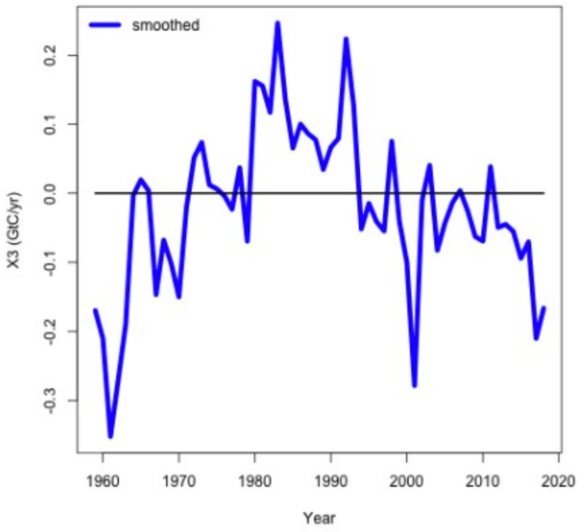
$X_{1,t}$ in C_t



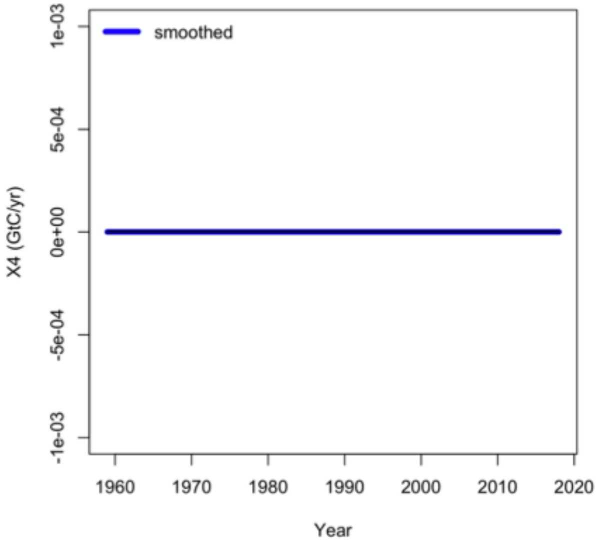
$X_{2,t}$ in S_LND_t



$X_{3,t}$ in S_OCN_t



$X_{4,t}$ in E_t



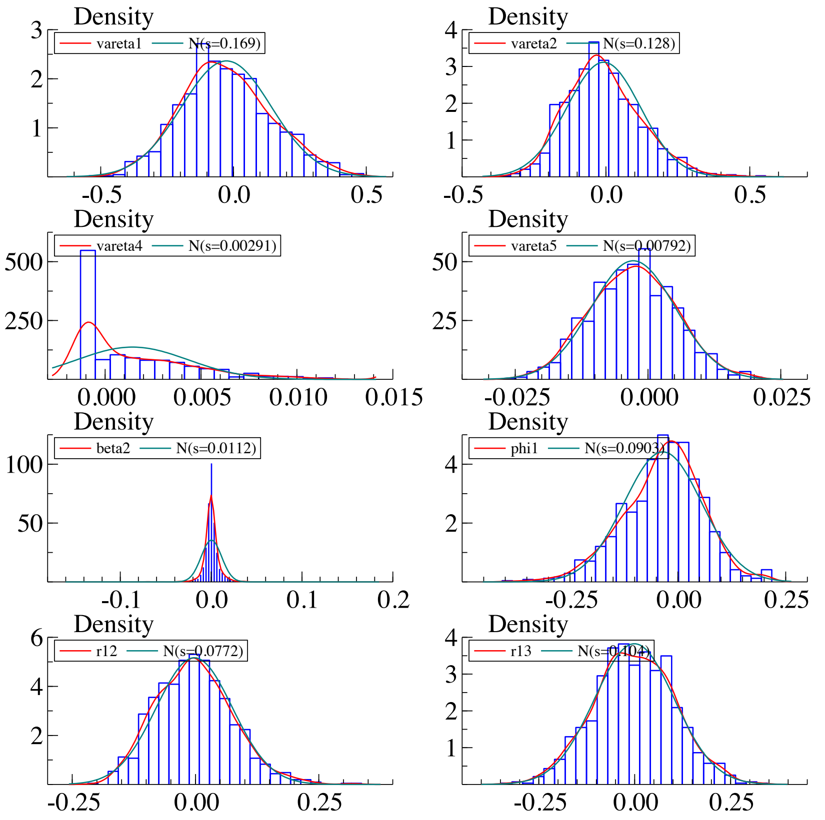
PARAMETER ESTIMATES

Model 2

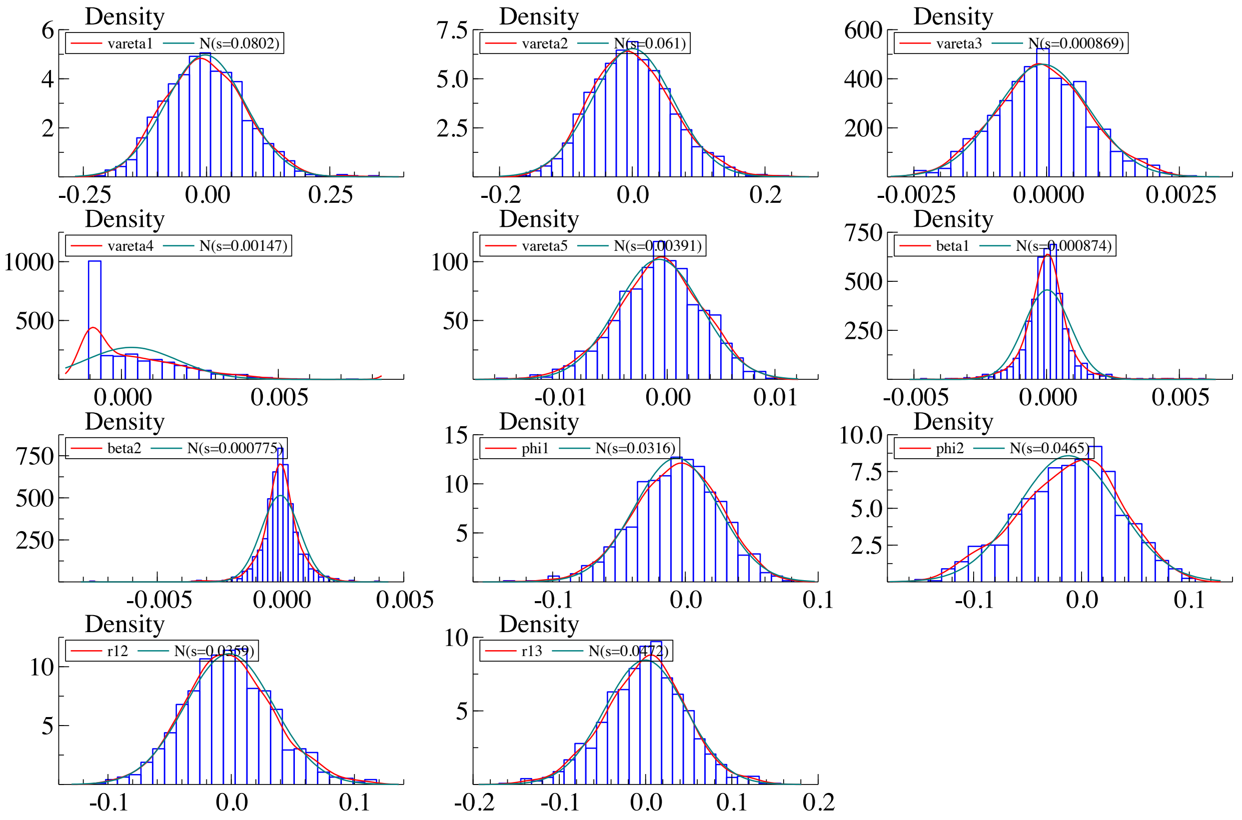
Coefficients	estimate	std err	Variances	estimate	std err
c_1 (filt.)	-6.77	0.05	$\sigma^2_{\eta_1}$	0.83	0.20
c_2 (filt.)	-5.35	0.04	$\sigma^2_{\eta_2}$	0.49	0.16
β_1	7.20	0.90	$\sigma^2_{\eta_3}$	0.008	0.002
β_2	5.57	0.48	$\sigma^2_{\eta_4}$	0.006	0.002
β_3 (filt.)	0.57	0.12	r_{12}	-0.63	0.15
β_4 (filt.)	-0.05	0.01	r_{13}	-0.08	0.13
β_5 (filt.)	3.15	0.30	s_E	2.38	0.57
β_6 (filt.)	-0.11	0.08			
β_7 (filt.)	-0.18	0.08			
β_8 (filt.)	-0.25	0.08			
β_9 (filt.)	-0.65	0.18			
ϕ_1	0.86	0.07			
ϕ_3	0.74	0.11			

SIMULATIONS OF MODEL 1

T=60

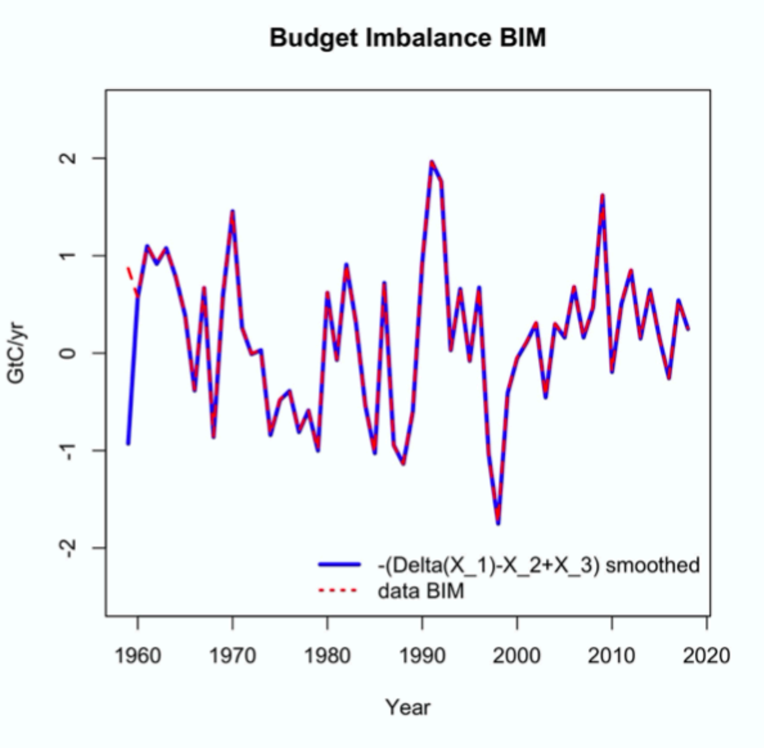


T=250

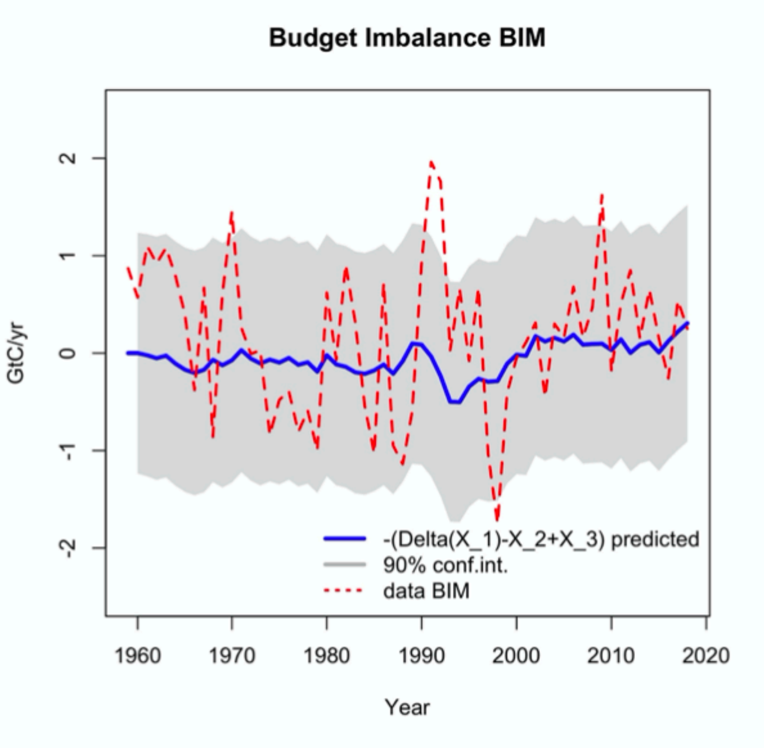


σ_4^2 exhibits “pile-up” problem (Stock and Watson 1998)

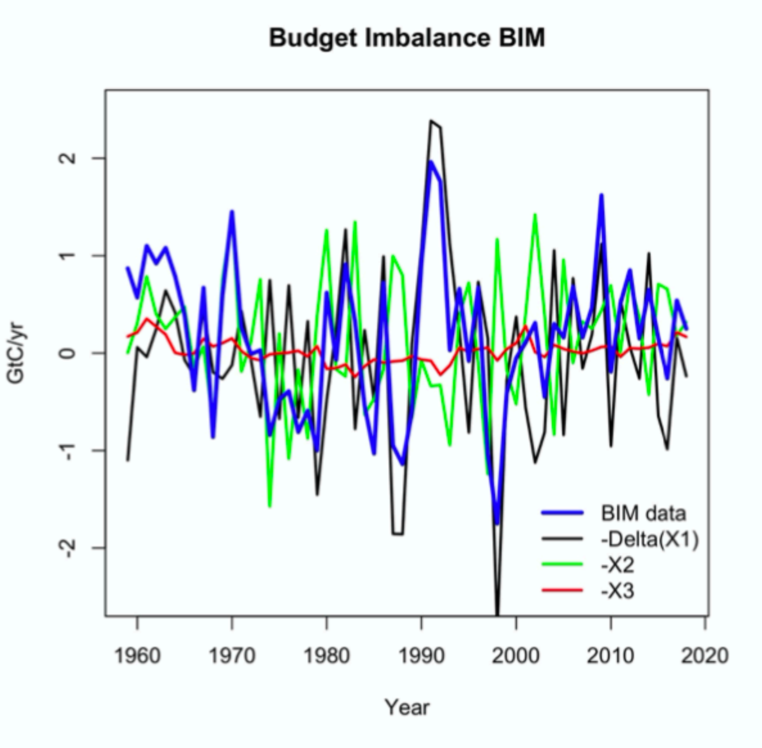
BUDGET IMBALANCE



(A) Smoothed $-(\Delta X_1 + X_2 + X_3)$

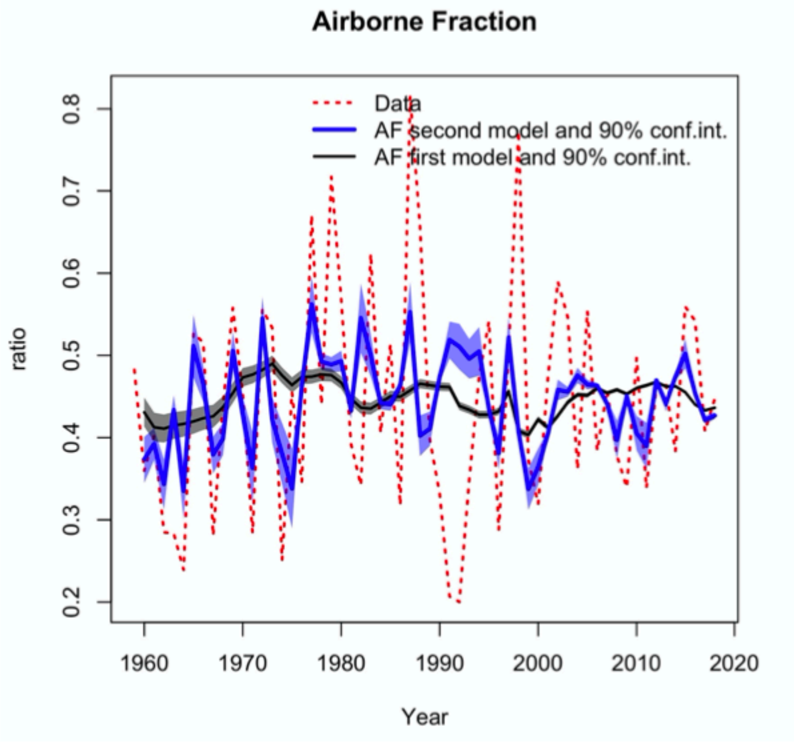


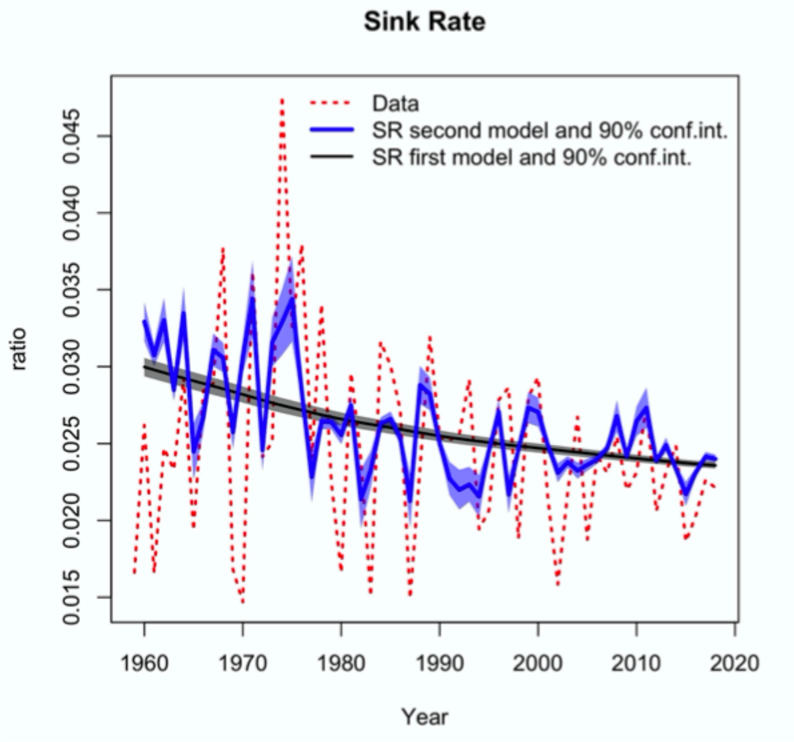
(B) One-year ahead predictions



(C) Components $-\Delta X_1, -X_2, -X_3$

AIRBORNE FRACTION AND SINK RATE





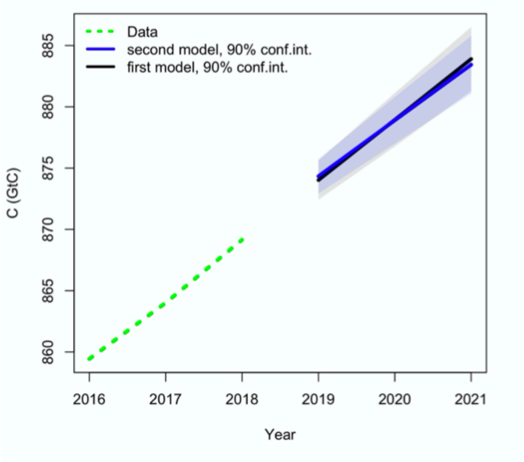
$$AF = \frac{\Delta C}{E}$$

$$SR = \frac{S\_LND + S\_OCN}{C}$$

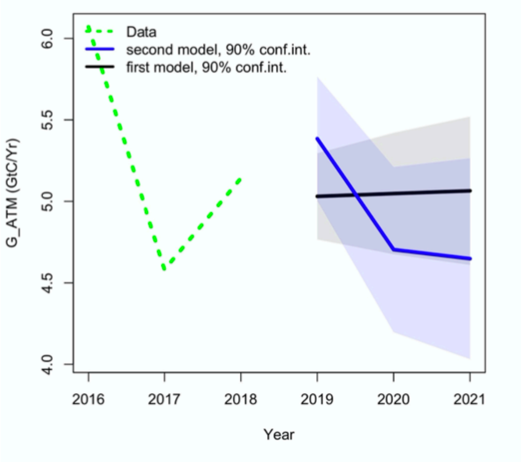
NOW-/FORECASTS

Forecasts of World GDP growth from IMF and World Bank

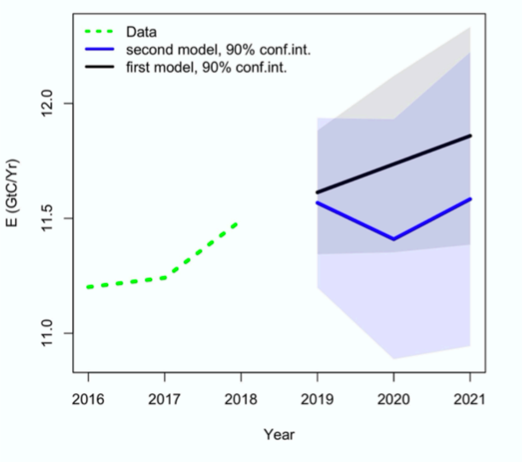
	2019	2020	2021
IMF	3.2%	-4.9%	5.4%
World Bank	2.6%	-5.2%	4.2%
Data	2.4%	N/A	N/A



(A) C

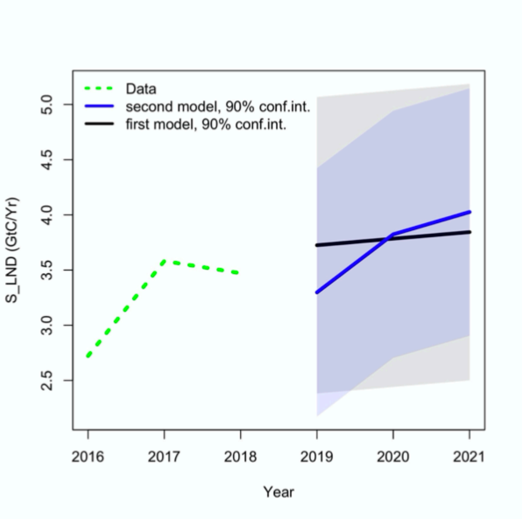


(B) G_ATM*

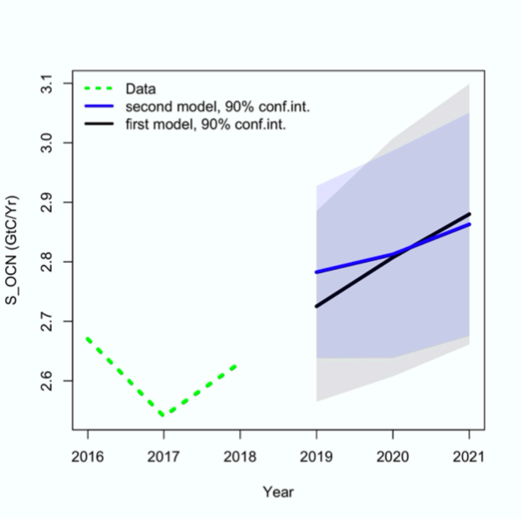


(C) E

Forecasts of SOI from forecast model of monthly SOI data 1866-1920, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years



(D) S_LND

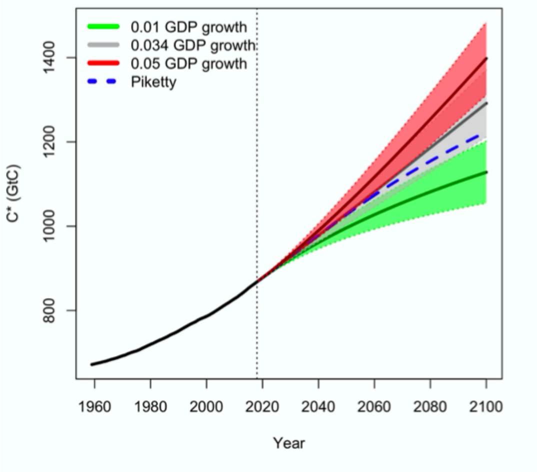


(E) S_OCN

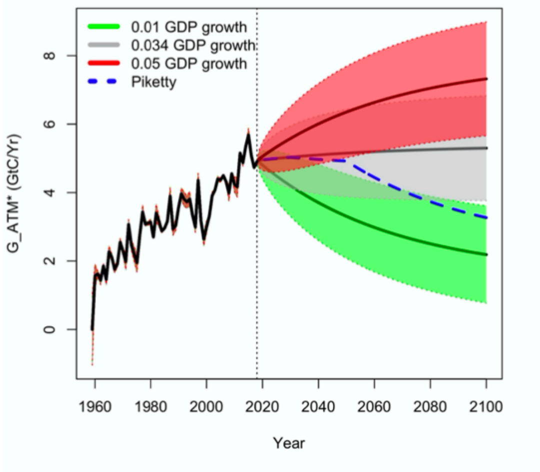
PROJECTIONS TO 2100

Scenarios:

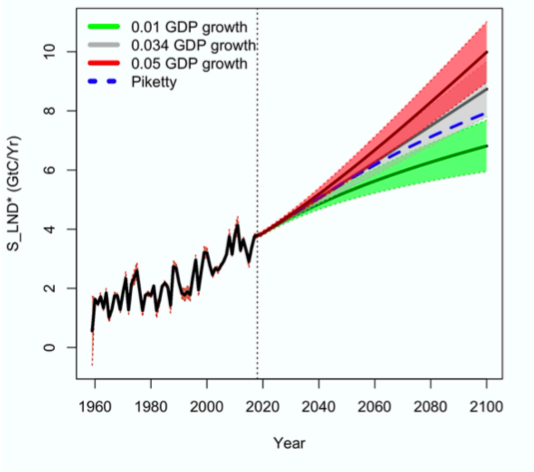
- 1% GDP growth
- 3.4% GDP growth
- 5% GDP growth
- Piketty scenario:
  - 3.4% til 2030
  - 3% til 2050
  - 1.5% til 2100



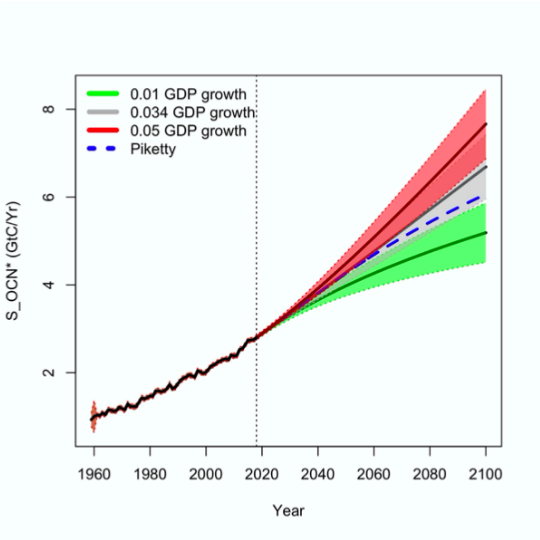
(A) C*



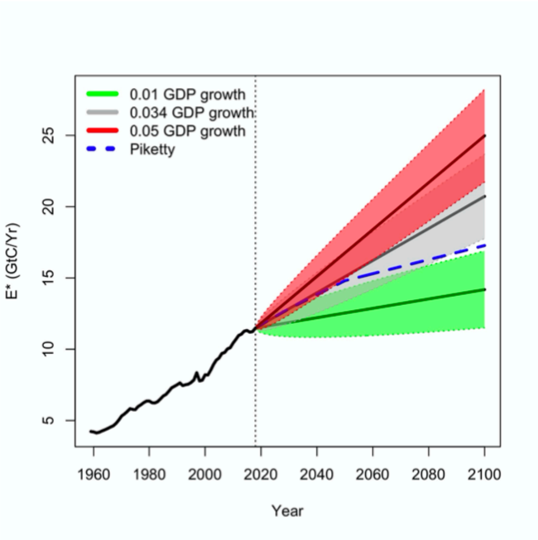
(B) G_ATM*



(C) S_LND*



(D) S_OCN*



(E) E*

CONCLUSIONS

- Specification of state-space model for Global Carbon Budget
- World GDP as driver in emissions
- Sinks: linear in CO2 concentrations and in SOI
- CO2 concentrations are I(1) ranging on I(2)
- Model allows for forecasting, projections, study of key variables such as airborne fraction and sink rate

Future directions

- Include ensemble members for S_LND and S_OCN
- Factor model for drift in emissions using large macroeconomic dataset
- Higher resolution on Global Carbon Cycle module (MAGICC)
- Connection to temperatures (Energy Balance Models)
- Cointegration analysis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